

Mathematica 11.3 Integration Test Results

Test results for the 111 problems in "6.2.2 (e x)^m (a+b x^n)^p cosh.m"

Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{\cosh[c + d x]}{x (a + b x)^3} dx$$

Optimal (type 4, 262 leaves, 17 steps) :

$$\begin{aligned} & \frac{\cosh[c + d x]}{2 a (a + b x)^2} + \frac{\cosh[c + d x]}{a^2 (a + b x)} + \frac{\cosh[c] \cosh\text{Integral}[d x]}{a^3} - \frac{\cosh[c - \frac{a d}{b}] \cosh\text{Integral}[\frac{a d}{b} + d x]}{a^3} - \\ & \frac{d^2 \cosh[c - \frac{a d}{b}] \cosh\text{Integral}[\frac{a d}{b} + d x]}{2 a b^2} - \frac{d \cosh\text{Integral}[\frac{a d}{b} + d x] \sinh[c - \frac{a d}{b}]}{a^2 b} + \\ & \frac{d \sinh[c + d x]}{2 a b (a + b x)} + \frac{\sinh[c] \sinh\text{Integral}[d x]}{a^3} - \frac{d \cosh[c - \frac{a d}{b}] \sinh\text{Integral}[\frac{a d}{b} + d x]}{a^2 b} - \\ & \frac{\sinh[c - \frac{a d}{b}] \sinh\text{Integral}[\frac{a d}{b} + d x]}{a^3} - \frac{d^2 \sinh[c - \frac{a d}{b}] \sinh\text{Integral}[\frac{a d}{b} + d x]}{2 a b^2} \end{aligned}$$

Result (type 4, 614 leaves) :

$$\begin{aligned}
& -\frac{1}{2 a^3 b^2 (a + b x)^2} \\
& \left(-3 a^2 b^2 \cosh[c + d x] - 2 a b^3 x \cosh[c + d x] - 2 b^2 (a + b x)^2 \cosh[c] \text{CoshIntegral}[d x] + \right. \\
& 2 b^2 (a + b x)^2 \cosh[c - \frac{a d}{b}] \text{CoshIntegral}\left[d \left(\frac{a}{b} + x\right)\right] + a^4 d^2 \cosh[c - \frac{a d}{b}] \\
& \text{CoshIntegral}\left[\frac{d (a + b x)}{b}\right] + 2 a^3 b d^2 x \cosh[c - \frac{a d}{b}] \text{CoshIntegral}\left[\frac{d (a + b x)}{b}\right] + \\
& a^2 b^2 d^2 x^2 \cosh[c - \frac{a d}{b}] \text{CoshIntegral}\left[\frac{d (a + b x)}{b}\right] + 2 a^3 b d \text{CoshIntegral}\left[\frac{d (a + b x)}{b}\right] \\
& \sinh[c - \frac{a d}{b}] + 4 a^2 b^2 d x \text{CoshIntegral}\left[\frac{d (a + b x)}{b}\right] \sinh[c - \frac{a d}{b}] + \\
& 2 a b^3 d x^2 \text{CoshIntegral}\left[\frac{d (a + b x)}{b}\right] \sinh[c - \frac{a d}{b}] - a^3 b d \sinh[c + d x] - \\
& a^2 b^2 d x \sinh[c + d x] - 2 a^2 b^2 \sinh[c] \text{SinhIntegral}[d x] - 4 a b^3 x \sinh[c] \text{SinhIntegral}[d x] - \\
& 2 b^4 x^2 \sinh[c] \text{SinhIntegral}[d x] + 2 a^2 b^2 \sinh[c - \frac{a d}{b}] \text{SinhIntegral}\left[d \left(\frac{a}{b} + x\right)\right] + \\
& 4 a b^3 x \sinh[c - \frac{a d}{b}] \text{SinhIntegral}\left[d \left(\frac{a}{b} + x\right)\right] + 2 b^4 x^2 \sinh[c - \frac{a d}{b}] \text{SinhIntegral}\left[d \left(\frac{a}{b} + x\right)\right] + \\
& 2 a^3 b d \cosh[c - \frac{a d}{b}] \text{SinhIntegral}\left[\frac{d (a + b x)}{b}\right] + 4 a^2 b^2 d x \cosh[c - \frac{a d}{b}] \\
& \text{SinhIntegral}\left[\frac{d (a + b x)}{b}\right] + 2 a b^3 d x^2 \cosh[c - \frac{a d}{b}] \text{SinhIntegral}\left[\frac{d (a + b x)}{b}\right] + \\
& a^4 d^2 \sinh[c - \frac{a d}{b}] \text{SinhIntegral}\left[\frac{d (a + b x)}{b}\right] + 2 a^3 b d^2 x \sinh[c - \frac{a d}{b}] \\
& \left. \text{SinhIntegral}\left[\frac{d (a + b x)}{b}\right] + a^2 b^2 d^2 x^2 \sinh[c - \frac{a d}{b}] \text{SinhIntegral}\left[\frac{d (a + b x)}{b}\right] \right)
\end{aligned}$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int \frac{\cosh[c + d x]}{x^2 (a + b x)^3} dx$$

Optimal (type 4, 298 leaves, 21 steps):

$$\begin{aligned}
& -\frac{\cosh[c + dx]}{a^3 x} - \frac{b \cosh[c + dx]}{2 a^2 (a + b x)^2} - \frac{2 b \cosh[c + dx]}{a^3 (a + b x)} - \\
& \frac{3 b \cosh[c] \operatorname{CoshIntegral}[dx]}{a^4} + \frac{3 b \cosh[c - \frac{ad}{b}] \operatorname{CoshIntegral}[\frac{ad}{b} + dx]}{a^4} + \\
& \frac{d^2 \cosh[c - \frac{ad}{b}] \operatorname{CoshIntegral}[\frac{ad}{b} + dx]}{2 a^2 b} + \frac{d \operatorname{CoshIntegral}[dx] \operatorname{Sinh}[c]}{a^3} + \\
& \frac{2 d \operatorname{CoshIntegral}[\frac{ad}{b} + dx] \operatorname{Sinh}[c - \frac{ad}{b}]}{a^3} - \frac{d \operatorname{Sinh}[c + dx]}{2 a^2 (a + b x)} + \frac{d \cosh[c] \operatorname{SinhIntegral}[dx]}{a^3} - \\
& \frac{3 b \operatorname{Sinh}[c] \operatorname{SinhIntegral}[dx]}{a^4} + \frac{2 d \cosh[c - \frac{ad}{b}] \operatorname{SinhIntegral}[\frac{ad}{b} + dx]}{a^3} + \\
& \frac{3 b \operatorname{Sinh}[c - \frac{ad}{b}] \operatorname{SinhIntegral}[\frac{ad}{b} + dx]}{a^4} + \frac{d^2 \operatorname{Sinh}[c - \frac{ad}{b}] \operatorname{SinhIntegral}[\frac{ad}{b} + dx]}{2 a^2 b}
\end{aligned}$$

Result (type 4, 710 leaves) :

$$\begin{aligned}
& \frac{1}{2 a^4 b x (a + b x)^2} \\
& \left(-2 a^3 b \cosh(c + d x) - 9 a^2 b^2 x \cosh(c + d x) - 6 a b^3 x^2 \cosh(c + d x) + 6 b^2 x (a + b x)^2 \right. \\
& \quad \cosh\left[c - \frac{a d}{b}\right] \text{CoshIntegral}\left[d \left(\frac{a}{b} + x\right)\right] + a^4 d^2 x \cosh\left[c - \frac{a d}{b}\right] \text{CoshIntegral}\left[\frac{d (a + b x)}{b}\right] + \\
& \quad 2 a^3 b d^2 x^2 \cosh\left[c - \frac{a d}{b}\right] \text{CoshIntegral}\left[\frac{d (a + b x)}{b}\right] + a^2 b^2 d^2 x^3 \cosh\left[c - \frac{a d}{b}\right] \\
& \quad \text{CoshIntegral}\left[\frac{d (a + b x)}{b}\right] + 2 b x (a + b x)^2 \text{CoshIntegral}[d x] (-3 b \cosh[c] + a d \sinh[c]) + \\
& \quad 4 a^3 b d x \text{CoshIntegral}\left[\frac{d (a + b x)}{b}\right] \sinh\left[c - \frac{a d}{b}\right] + 8 a^2 b^2 d x^2 \text{CoshIntegral}\left[\frac{d (a + b x)}{b}\right] \\
& \quad \sinh\left[c - \frac{a d}{b}\right] + 4 a b^3 d x^3 \text{CoshIntegral}\left[\frac{d (a + b x)}{b}\right] \sinh\left[c - \frac{a d}{b}\right] - \\
& \quad a^3 b d x \sinh(c + d x) - a^2 b^2 d x^2 \sinh(c + d x) + 2 a^3 b d x \cosh[c] \text{SinhIntegral}[d x] + \\
& \quad 4 a^2 b^2 d x^2 \cosh[c] \text{SinhIntegral}[d x] + 2 a b^3 d x^3 \cosh[c] \text{SinhIntegral}[d x] - \\
& \quad 6 a^2 b^2 x \sinh[c] \text{SinhIntegral}[d x] - 12 a b^3 x^2 \sinh[c] \text{SinhIntegral}[d x] - \\
& \quad 6 b^4 x^3 \sinh[c] \text{SinhIntegral}[d x] + 6 a^2 b^2 x \sinh\left[c - \frac{a d}{b}\right] \text{SinhIntegral}\left[d \left(\frac{a}{b} + x\right)\right] + \\
& \quad 12 a b^3 x^2 \sinh\left[c - \frac{a d}{b}\right] \text{SinhIntegral}\left[d \left(\frac{a}{b} + x\right)\right] + \\
& \quad 6 b^4 x^3 \sinh\left[c - \frac{a d}{b}\right] \text{SinhIntegral}\left[d \left(\frac{a}{b} + x\right)\right] + \\
& \quad 4 a^3 b d x \cosh\left[c - \frac{a d}{b}\right] \text{SinhIntegral}\left[\frac{d (a + b x)}{b}\right] + 8 a^2 b^2 d x^2 \cosh\left[c - \frac{a d}{b}\right] \\
& \quad \text{SinhIntegral}\left[\frac{d (a + b x)}{b}\right] + 4 a b^3 d x^3 \cosh\left[c - \frac{a d}{b}\right] \text{SinhIntegral}\left[\frac{d (a + b x)}{b}\right] + \\
& \quad a^4 d^2 x \sinh\left[c - \frac{a d}{b}\right] \text{SinhIntegral}\left[\frac{d (a + b x)}{b}\right] + 2 a^3 b d^2 x^2 \sinh\left[c - \frac{a d}{b}\right] \\
& \quad \text{SinhIntegral}\left[\frac{d (a + b x)}{b}\right] + a^2 b^2 d^2 x^3 \sinh\left[c - \frac{a d}{b}\right] \text{SinhIntegral}\left[\frac{d (a + b x)}{b}\right] \Big)
\end{aligned}$$

Problem 57: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 \cosh(c + d x)}{a + b x^2} dx$$

Optimal (type 4, 273 leaves, 14 steps):

$$\begin{aligned}
& -\frac{2 x \cosh[c + d x]}{b d^2} + \frac{(-a)^{3/2} \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \text{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{2 b^{5/2}} - \\
& \frac{(-a)^{3/2} \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \text{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{2 b^{5/2}} + \frac{2 \sinh[c + d x]}{b d^3} - \frac{a \sinh[c + d x]}{b^2 d} + \\
& \frac{x^2 \sinh[c + d x]}{b d} - \frac{(-a)^{3/2} \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \text{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{2 b^{5/2}} - \\
& \frac{(-a)^{3/2} \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \text{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{2 b^{5/2}}
\end{aligned}$$

Result (type 4, 274 leaves):

$$\begin{aligned}
& \frac{1}{2 b^{5/2} d^3} \\
& \left(-4 b^{3/2} d x \cosh[c + d x] + i a^{3/2} d^3 \cosh[c - \frac{i \sqrt{a} d}{\sqrt{b}}] \text{CosIntegral}[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x] - i a^{3/2} d^3 \right. \\
& \quad \cosh[c + \frac{i \sqrt{a} d}{\sqrt{b}}] \text{CosIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] + 4 b^{3/2} \sinh[c + d x] - 2 a \sqrt{b} d^2 \sinh[c + d x] + \\
& \quad 2 b^{3/2} d^2 x^2 \sinh[c + d x] - a^{3/2} d^3 \sinh[c - \frac{i \sqrt{a} d}{\sqrt{b}}] \text{SinIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} - i d x] - \\
& \quad \left. a^{3/2} d^3 \sinh[c + \frac{i \sqrt{a} d}{\sqrt{b}}] \text{SinIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \right)
\end{aligned}$$

Problem 58: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3 \cosh[c + d x]}{a + b x^2} dx$$

Optimal (type 4, 209 leaves, 12 steps):

$$\begin{aligned}
& -\frac{\cosh[c + d x]}{b d^2} - \frac{a \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \text{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{2 b^2} - \\
& \frac{a \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \text{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{2 b^2} + \frac{x \sinh[c + d x]}{b d} + \\
& \frac{a \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \text{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{2 b^2} - \frac{a \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \text{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{2 b^2}
\end{aligned}$$

Result (type 4, 210 leaves):

$$\begin{aligned}
& -\frac{1}{2 b^2 d^2} \left(2 b \cosh[c + d x] + a d^2 \cosh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{CosIntegral}\left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] + \right. \\
& \quad a d^2 \cosh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{CosIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] - 2 b d x \sinh[c + d x] + \\
& \quad i a d^2 \sinh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] - \\
& \quad \left. i a d^2 \sinh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \right)
\end{aligned}$$

Problem 59: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 \cosh[c + d x]}{a + b x^2} dx$$

Optimal (type 4, 226 leaves, 11 steps):

$$\begin{aligned}
& \frac{\sqrt{-a} \cosh\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{2 b^{3/2}} - \\
& \frac{\sqrt{-a} \cosh\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{2 b^{3/2}} + \\
& \frac{\sinh[c + d x]}{b d} - \frac{\sqrt{-a} \sinh\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{2 b^{3/2}} - \\
& \frac{\sqrt{-a} \sinh\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \text{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{2 b^{3/2}}
\end{aligned}$$

Result (type 4, 213 leaves):

$$\begin{aligned}
& \frac{1}{2 b^{3/2} d} \left(-i \sqrt{a} d \cosh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{CosIntegral}\left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] + \right. \\
& \quad i \sqrt{a} d \cosh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{CosIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] + \\
& \quad 2 \sqrt{b} \sinh[c + d x] + \sqrt{a} d \sinh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] + \\
& \quad \left. \sqrt{a} d \sinh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \right)
\end{aligned}$$

Problem 60: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x \cosh[c + d x]}{a + b x^2} dx$$

Optimal (type 4, 177 leaves, 8 steps) :

$$\frac{\cosh\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \cosh\text{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{2 b} + \frac{\cosh\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \cosh\text{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{2 b} -$$

$$\frac{\sinh\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \sinh\text{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{2 b} + \frac{\sinh\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \sinh\text{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{2 b}$$

Result (type 4, 171 leaves) :

$$\frac{1}{2 b} \left(\cosh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \cos\text{Integral}\left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] + \cosh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \cos\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] + i \left(\sinh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \sin\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] - \sinh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \sin\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \right) \right)$$

Problem 61: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cosh[c + d x]}{a + b x^2} dx$$

Optimal (type 4, 213 leaves, 8 steps) :

$$\frac{\cosh\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \cosh\text{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{2 \sqrt{-a} \sqrt{b}} - \frac{\cosh\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \cosh\text{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{2 \sqrt{-a} \sqrt{b}} -$$

$$\frac{\sinh\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \sinh\text{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{2 \sqrt{-a} \sqrt{b}} - \frac{\sinh\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \sinh\text{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{2 \sqrt{-a} \sqrt{b}}$$

Result (type 4, 180 leaves) :

$$\frac{1}{2 \sqrt{a} \sqrt{b}}$$

$$i \left(\cosh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \cos\text{Integral}\left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] - \cosh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \cos\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] + i \left(\sinh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \sin\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] + \sinh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \sin\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \right) \right)$$

Problem 62: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cosh[c + d x]}{x (a + b x^2)} dx$$

Optimal (type 4, 197 leaves, 13 steps):

$$\begin{aligned} & \frac{\cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \cosh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{a} - \frac{\cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \cosh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{2 a} \\ & + \frac{\sinh[c] \sinh\text{Integral}[d x]}{a} + \frac{\sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{2 a} - \frac{\sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{2 a} \end{aligned}$$

Result (type 4, 187 leaves):

$$\begin{aligned} & -\frac{1}{2 a} \left(-2 \cosh[c] \cosh\text{Integral}[d x] + \right. \\ & \cosh[c - \frac{i \sqrt{a} d}{\sqrt{b}}] \cos\text{Integral}[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x] + \cosh[c + \frac{i \sqrt{a} d}{\sqrt{b}}] \cos\text{Integral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] - \\ & 2 \sinh[c] \sinh\text{Integral}[d x] + i \sinh[c - \frac{i \sqrt{a} d}{\sqrt{b}}] \sin\text{Integral}[\frac{\sqrt{a} d}{\sqrt{b}} - i d x] - \\ & \left. i \sinh[c + \frac{i \sqrt{a} d}{\sqrt{b}}] \sin\text{Integral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \right) \end{aligned}$$

Problem 63: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cosh[c + d x]}{x^2 (a + b x^2)} dx$$

Optimal (type 4, 249 leaves, 14 steps):

$$\begin{aligned} & -\frac{\cosh[c + d x]}{a x} + \frac{\sqrt{b} \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \cosh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{2 (-a)^{3/2}} - \\ & \frac{\sqrt{b} \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \cosh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{2 (-a)^{3/2}} + \frac{d \cosh\text{Integral}[d x] \sinh[c]}{a} + \\ & \frac{d \cosh[c] \sinh\text{Integral}[d x]}{a} - \frac{\sqrt{b} \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{2 (-a)^{3/2}} - \\ & \frac{\sqrt{b} \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{2 (-a)^{3/2}} \end{aligned}$$

Result (type 4, 243 leaves):

$$\frac{1}{2 a^{3/2} x} \left(-2 \sqrt{a} \cosh[c + d x] - i \sqrt{b} \times \cosh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{CosIntegral}\left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] + \right.$$

$$i \sqrt{b} \times \cosh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{CosIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] + 2 \sqrt{a} d x \cosh\text{Integral}[d x] \sinh[c] +$$

$$2 \sqrt{a} d x \cosh[c] \sinh\text{Integral}[d x] + \sqrt{b} \times \sinh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] +$$

$$\left. \sqrt{b} \times \sinh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \right)$$

Problem 64: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cosh[c + d x]}{x^3 (a + b x^2)} dx$$

Optimal (type 4, 270 leaves, 18 steps):

$$\begin{aligned} & -\frac{\cosh[c + d x]}{2 a^2 x^2} - \frac{b \cosh[c] \cosh\text{Integral}[d x]}{a^2} + \frac{d^2 \cosh[c] \cosh\text{Integral}[d x]}{2 a} + \\ & \frac{b \cosh\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \cosh\text{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{2 a^2} + \frac{b \cosh\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \cosh\text{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{2 a^2} - \\ & \frac{d \sinh[c + d x]}{2 a x} - \frac{b \sinh[c] \sinh\text{Integral}[d x]}{a^2} + \frac{d^2 \sinh[c] \sinh\text{Integral}[d x]}{2 a} - \\ & \frac{b \sinh\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \sinh\text{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{2 a^2} + \frac{b \sinh\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \sinh\text{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{2 a^2} \end{aligned}$$

Result (type 4, 257 leaves):

$$\begin{aligned} & \frac{1}{2 a^2 x^2} \left(-a \cosh[c + d x] - (2 b - a d^2) x^2 \cosh[c] \cosh\text{Integral}[d x] + \right. \\ & b x^2 \cosh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{CosIntegral}\left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] + \\ & b x^2 \cosh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{CosIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] - a d x \sinh[c + d x] - \\ & 2 b x^2 \sinh[c] \sinh\text{Integral}[d x] + a d^2 x^2 \sinh[c] \sinh\text{Integral}[d x] + \\ & i b x^2 \sinh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] - \\ & \left. i b x^2 \sinh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \right) \end{aligned}$$

Problem 65: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 \cosh[c + d x]}{(a + b x^2)^2} dx$$

Optimal (type 4, 449 leaves, 24 steps):

$$\begin{aligned} & \frac{x \cosh[c + d x]}{2 b^2} - \frac{x^3 \cosh[c + d x]}{2 b (a + b x^2)} + \frac{3 \sqrt{-a} \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \text{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{4 b^{5/2}} - \\ & \frac{3 \sqrt{-a} \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \text{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{4 b^{5/2}} - \\ & \frac{a d \text{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x] \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}]}{4 b^3} - \frac{a d \text{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x] \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}]}{4 b^3} + \\ & \frac{\sinh[c + d x]}{b^2 d} + \frac{a d \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \text{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{4 b^3} - \\ & \frac{3 \sqrt{-a} \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \text{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{4 b^{5/2}} - \\ & \frac{a d \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \text{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{4 b^3} - \frac{3 \sqrt{-a} \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \text{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{4 b^{5/2}} \end{aligned}$$

Result (type 4, 621 leaves):

$$\begin{aligned}
& \frac{1}{4 b^2} \left(2 \operatorname{Cosh}[d x] \left(\frac{a x \operatorname{Cosh}[c]}{a + b x^2} + \frac{2 \operatorname{Sinh}[c]}{d} \right) + 2 \left(\frac{2 \operatorname{Cosh}[c]}{d} + \frac{a x \operatorname{Sinh}[c]}{a + b x^2} \right) \operatorname{Sinh}[d x] - \frac{1}{\sqrt{b}} 3 i \sqrt{a} \right. \\
& \operatorname{Cosh}[c] \left(\operatorname{Cos} \left[\frac{\sqrt{a} d}{\sqrt{b}} \right] \operatorname{CosIntegral} \left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] - \operatorname{Cos} \left[\frac{\sqrt{a} d}{\sqrt{b}} \right] \operatorname{CosIntegral} \left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] + \right. \\
& \sin \left[\frac{\sqrt{a} d}{\sqrt{b}} \right] \left(\operatorname{SinIntegral} \left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x \right] - \operatorname{SinIntegral} \left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] \right) + \frac{1}{b} i a d \operatorname{Cosh}[c] \\
& \left(\operatorname{CosIntegral} \left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] \sin \left[\frac{\sqrt{a} d}{\sqrt{b}} \right] - \operatorname{CosIntegral} \left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] \sin \left[\frac{\sqrt{a} d}{\sqrt{b}} \right] + \right. \\
& \cos \left[\frac{\sqrt{a} d}{\sqrt{b}} \right] \left(-\operatorname{SinIntegral} \left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x \right] + \operatorname{SinIntegral} \left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] \right) - \frac{1}{\sqrt{b}} 3 \sqrt{a} \\
& \operatorname{Sinh}[c] \left(\operatorname{CosIntegral} \left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] \sin \left[\frac{\sqrt{a} d}{\sqrt{b}} \right] + \operatorname{CosIntegral} \left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] \sin \left[\frac{\sqrt{a} d}{\sqrt{b}} \right] - \right. \\
& \cos \left[\frac{\sqrt{a} d}{\sqrt{b}} \right] \left(\operatorname{SinIntegral} \left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x \right] + \operatorname{SinIntegral} \left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] \right) - \frac{1}{b} a d \operatorname{Sinh}[c] \\
& \left(\cos \left[\frac{\sqrt{a} d}{\sqrt{b}} \right] \operatorname{CosIntegral} \left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] + \cos \left[\frac{\sqrt{a} d}{\sqrt{b}} \right] \operatorname{CosIntegral} \left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] + \right. \\
& \left. \sin \left[\frac{\sqrt{a} d}{\sqrt{b}} \right] \left(\operatorname{SinIntegral} \left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x \right] + \operatorname{SinIntegral} \left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] \right) \right)
\end{aligned}$$

Problem 66: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3 \operatorname{Cosh}[c + d x]}{(a + b x^2)^2} dx$$

Optimal (type 4, 431 leaves, 20 steps):

$$\begin{aligned}
& \frac{\operatorname{Cosh}[c + d x]}{2 b^2} - \frac{x^2 \operatorname{Cosh}[c + d x]}{2 b (a + b x^2)} + \frac{\operatorname{Cosh} \left[c + \frac{\sqrt{-a} d}{\sqrt{b}} \right] \operatorname{CoshIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x \right]}{2 b^2} + \\
& \frac{\operatorname{Cosh} \left[c - \frac{\sqrt{-a} d}{\sqrt{b}} \right] \operatorname{CoshIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x \right]}{2 b^2} - \frac{\sqrt{-a} d \operatorname{CoshIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x \right] \operatorname{Sinh} \left[c - \frac{\sqrt{-a} d}{\sqrt{b}} \right]}{4 b^{5/2}} + \\
& \frac{\sqrt{-a} d \operatorname{CoshIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x \right] \operatorname{Sinh} \left[c + \frac{\sqrt{-a} d}{\sqrt{b}} \right]}{4 b^{5/2}} - \\
& \frac{\sqrt{-a} d \operatorname{Cosh} \left[c + \frac{\sqrt{-a} d}{\sqrt{b}} \right] \operatorname{SinhIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x \right]}{4 b^{5/2}} - \frac{\operatorname{Sinh} \left[c + \frac{\sqrt{-a} d}{\sqrt{b}} \right] \operatorname{SinhIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x \right]}{2 b^2} - \\
& \frac{\sqrt{-a} d \operatorname{Cosh} \left[c - \frac{\sqrt{-a} d}{\sqrt{b}} \right] \operatorname{SinhIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x \right]}{4 b^{5/2}} + \frac{\operatorname{Sinh} \left[c - \frac{\sqrt{-a} d}{\sqrt{b}} \right] \operatorname{SinhIntegral} \left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x \right]}{2 b^2}
\end{aligned}$$

Result (type 4, 582 leaves):

$$\begin{aligned}
& \frac{1}{4 b^{5/2} (a + b x^2)} \left(2 a \sqrt{b} \cosh[c + d x] + \right. \\
& (a + b x^2) \operatorname{CosIntegral}\left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] \left(2 \sqrt{b} \cosh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] - i \sqrt{a} d \sinh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \right) + \\
& (a + b x^2) \operatorname{CosIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] \left(2 \sqrt{b} \cosh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] + i \sqrt{a} d \sinh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \right) + \\
& a^{3/2} d \cosh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x \right] + \\
& \sqrt{a} b d x^2 \cosh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x \right] + \\
& 2 i a \sqrt{b} \sinh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x \right] + \\
& 2 i b^{3/2} x^2 \sinh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x \right] + \\
& a^{3/2} d \cosh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] + \\
& \sqrt{a} b d x^2 \cosh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] - \\
& 2 i a \sqrt{b} \sinh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] - \\
& \left. 2 i b^{3/2} x^2 \sinh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] \right)
\end{aligned}$$

Problem 67: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 \cosh[c + d x]}{(a + b x^2)^2} dx$$

Optimal (type 4, 416 leaves, 17 steps):

$$\begin{aligned}
& -\frac{x \cosh[c + d x]}{2 b (a + b x^2)} + \frac{\cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{4 \sqrt{-a} b^{3/2}} - \\
& \frac{\cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{4 \sqrt{-a} b^{3/2}} + \frac{d \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x] \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}]}{4 b^2} + \\
& \frac{d \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x] \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}]}{4 b^2} - \\
& \frac{d \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{4 b^2} - \frac{\sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{4 \sqrt{-a} b^{3/2}} + \\
& \frac{d \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{4 b^2} - \frac{\sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{4 \sqrt{-a} b^{3/2}}
\end{aligned}$$

Result (type 4, 364 leaves):

$$\begin{aligned}
& \frac{1}{4 \sqrt{a} b^2 (a + b x^2)} \left(-2 \sqrt{a} b x \cosh[c + d x] + \right. \\
& (a + b x^2) \operatorname{CosIntegral}[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \left(i \sqrt{b} \cosh[c - \frac{i \sqrt{a} d}{\sqrt{b}}] + \sqrt{a} d \sinh[c - \frac{i \sqrt{a} d}{\sqrt{b}}] \right) + \\
& (a + b x^2) \operatorname{CosIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \left(-i \sqrt{b} \cosh[c + \frac{i \sqrt{a} d}{\sqrt{b}}] + \sqrt{a} d \sinh[c + \frac{i \sqrt{a} d}{\sqrt{b}}] \right) + \\
& (a + b x^2) \left(i \sqrt{a} d \cosh[c - \frac{i \sqrt{a} d}{\sqrt{b}}] - \sqrt{b} \sinh[c - \frac{i \sqrt{a} d}{\sqrt{b}}] \right) \operatorname{SinIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} - i d x] - \\
& \left. (a + b x^2) \left(i \sqrt{a} d \cosh[c + \frac{i \sqrt{a} d}{\sqrt{b}}] + \sqrt{b} \sinh[c + \frac{i \sqrt{a} d}{\sqrt{b}}] \right) \operatorname{SinIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \right)
\end{aligned}$$

Problem 68: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x \cosh[c + d x]}{(a + b x^2)^2} dx$$

Optimal (type 4, 239 leaves, 9 steps):

$$\begin{aligned}
& -\frac{\cosh[c + d x]}{2 b (a + b x^2)} - \frac{d \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x] \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}]}{4 \sqrt{-a} b^{3/2}} + \\
& \frac{d \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x] \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}]}{4 \sqrt{-a} b^{3/2}} - \\
& \frac{d \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{4 \sqrt{-a} b^{3/2}} - \frac{d \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{4 \sqrt{-a} b^{3/2}}
\end{aligned}$$

Result (type 4, 239 leaves) :

$$\begin{aligned} & \left(\frac{i}{\sqrt{b}} \left(d (a + b x^2) \operatorname{CosIntegral}\left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \operatorname{Sinh}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] - \right. \right. \\ & d (a + b x^2) \operatorname{CosIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \operatorname{Sinh}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] + \\ & \left. \left. \frac{i}{2 \sqrt{a} \sqrt{b}} \cosh[c + d x] + d (a + b x^2) \cosh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] + \right. \right. \\ & \left. \left. d (a + b x^2) \cosh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \right) \right) / \left(4 \sqrt{a} b^{3/2} (a + b x^2) \right) \end{aligned}$$

Problem 69: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cosh[c + d x]}{(a + b x^2)^2} dx$$

Optimal (type 4, 476 leaves, 18 steps) :

$$\begin{aligned} & -\frac{\cosh[c + d x]}{4 a \sqrt{b} (\sqrt{-a} - \sqrt{b} x)} + \frac{\cosh[c + d x]}{4 a \sqrt{b} (\sqrt{-a} + \sqrt{b} x)} - \\ & \frac{\cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \cosh\operatorname{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{4 (-a)^{3/2} \sqrt{b}} + \frac{\cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \cosh\operatorname{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{4 (-a)^{3/2} \sqrt{b}} - \\ & \frac{d \cosh\operatorname{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right] \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}]}{4 a b} - \frac{d \cosh\operatorname{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right] \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}]}{4 a b} + \\ & \frac{d \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\operatorname{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{4 a b} + \frac{\sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\operatorname{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{4 (-a)^{3/2} \sqrt{b}} - \\ & \frac{d \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\operatorname{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{4 a b} + \frac{\sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\operatorname{Integral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{4 (-a)^{3/2} \sqrt{b}} \end{aligned}$$

Result (type 4, 590 leaves) :

$$\begin{aligned}
& \frac{1}{4 a^{3/2} b (a + b x^2)} \left(2 \sqrt{a} b x \cosh[c + d x] - \right. \\
& (a + b x^2) \operatorname{CosIntegral}\left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] \left(-i \sqrt{b} \cosh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] + \sqrt{a} d \sinh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \right) - \\
& (a + b x^2) \operatorname{CosIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] \left(i \sqrt{b} \cosh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] + \sqrt{a} d \sinh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \right) - \\
& i a^{3/2} d \cosh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x \right] - \\
& i \sqrt{a} b d x^2 \cosh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x \right] - \\
& a \sqrt{b} \sinh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x \right] - \\
& b^{3/2} x^2 \sinh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x \right] + \\
& i a^{3/2} d \cosh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] + \\
& i \sqrt{a} b d x^2 \cosh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] - \\
& a \sqrt{b} \sinh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] - \\
& \left. b^{3/2} x^2 \sinh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x \right] \right)
\end{aligned}$$

Problem 70: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\cosh[c + d x]}{x (a + b x^2)^2} dx$$

Optimal (type 4, 435 leaves, 22 steps):

$$\begin{aligned}
& \frac{\cosh[c + d x]}{2 a (a + b x^2)} + \frac{\cosh[c] \cosh\text{Integral}[d x]}{a^2} - \frac{\cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \cosh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{2 a^2} - \\
& \frac{\cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \cosh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{2 a^2} - \frac{d \cosh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x] \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}]}{4 (-a)^{3/2} \sqrt{b}} + \\
& \frac{d \cosh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x] \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}]}{4 (-a)^{3/2} \sqrt{b}} + \frac{\sinh[c] \sinh\text{Integral}[d x]}{a^2} - \\
& \frac{d \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{4 (-a)^{3/2} \sqrt{b}} + \frac{\sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{2 a^2} - \\
& \frac{d \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{4 (-a)^{3/2} \sqrt{b}} - \frac{\sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{2 a^2}
\end{aligned}$$

Result (type 4, 2464 leaves):

$$\begin{aligned}
& \sinh[c] \left(\frac{\sinh\text{Integral}[d x]}{a^2} - \frac{1}{2 a^2} \right. \\
& \left. - i \cosh\text{Integral}[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)] \sin[\frac{\sqrt{a} d}{\sqrt{b}}] + \cos[\frac{\sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)] \right) - \\
& \frac{1}{4 a^{3/2} i \sqrt{b}} \left(- \frac{\sinh[d x]}{i \sqrt{a} \sqrt{b} + b x} + \frac{1}{b} d \left(\cos[\frac{\sqrt{a} d}{\sqrt{b}}] \cosh\text{Integral}[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)] \right. \right. \\
& \left. \left. - i \sin[\frac{\sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)] \right) + \frac{1}{2 a^2} \right. \\
& \left. - i \cosh\text{Integral}[-\frac{i \sqrt{a} d}{\sqrt{b}} + d x] \sin[\frac{\sqrt{a} d}{\sqrt{b}}] + \cos[\frac{\sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{i \sqrt{a} d}{\sqrt{b}} - d x] \right) + \\
& \frac{1}{4 a^{3/2} i \sqrt{b}} \left(- \frac{\sinh[d x]}{-i \sqrt{a} \sqrt{b} + b x} + \frac{1}{b} d \left(\cos[\frac{\sqrt{a} d}{\sqrt{b}}] \cosh\text{Integral}[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x \right)] \right. \right. \\
& \left. \left. - i \sin[\frac{\sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{i \sqrt{a} d}{\sqrt{b}} - d x] \right) \right) + \cosh[c] \\
& \left(\frac{\cosh\text{Integral}[d x]}{a^2} - \frac{1}{4 a^{3/2} i \sqrt{b}} \left(- \frac{\cosh[d x]}{i \sqrt{a} \sqrt{b} + b x} + \frac{1}{b} d \left(-i \cosh\text{Integral}[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)] \right. \right. \right. \\
& \left. \left. \left. \sin[\frac{\sqrt{a} d}{\sqrt{b}}] + \cos[\frac{\sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)] \right) \right) - \frac{1}{2 a^2} \right. \\
& \left. \left(\cos[\frac{\sqrt{a} d}{\sqrt{b}}] \cosh\text{Integral}[-\frac{i \sqrt{a} d}{\sqrt{b}} + d x] - i \sin[\frac{\sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{i \sqrt{a} d}{\sqrt{b}} - d x] \right) + \right. \\
& \left. \left. \frac{1}{4 a^{3/2} i \sqrt{b}} \left(- \frac{\cosh[d x]}{-i \sqrt{a} \sqrt{b} + b x} - \frac{1}{b} d \left(-i \cosh\text{Integral}[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x \right)] \right. \right. \right. \right. \\
& \left. \left. \left. \left. \sin[\frac{\sqrt{a} d}{\sqrt{b}}] \right) \right. \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{i}{\sqrt{b}} \operatorname{Sin}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[d\left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right]\right)\left.\right)+\frac{1}{2 a^2}\left(-\frac{i}{\sqrt{b}}\right. \\
& \left.\operatorname{CoshIntegral}\left[-\frac{i \sqrt{a} d}{\sqrt{b}}+d x\right] \operatorname{Sin}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right]+\operatorname{Cos}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}}-d x\right]\right)+ \\
& \frac{1}{4 a^{3/2}} \frac{i}{\sqrt{b}}\left(-\frac{\operatorname{Sinh}[d x]}{-\frac{i}{\sqrt{a}} \sqrt{b}+b x}+\frac{1}{b} d\left(\operatorname{Cos}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[d\left(-\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right]-\right.\right. \\
& \left.\left.\frac{i}{\sqrt{b}} \operatorname{Sin}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}}-d x\right]\right)\right)+ \\
& \operatorname{Sinh}[c]\left(\frac{\operatorname{CoshIntegral}[d x]}{a^2}-\frac{1}{4 a^{3/2}} \frac{i}{\sqrt{b}}\left(-\frac{\operatorname{Cosh}[d x]}{-\frac{i}{\sqrt{a}} \sqrt{b}+b x}+\frac{1}{b} d\left(-\frac{i}{\sqrt{a}} \operatorname{CoshIntegral}[d\right.\right. \\
& \left.\left.\left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right] \operatorname{Sin}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right]+\operatorname{Cos}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[d\left(\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right]\right)-\frac{1}{2 a^2}\right. \\
& \left.\left(\operatorname{Cos}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[-\frac{i \sqrt{a} d}{\sqrt{b}}+d x\right]-\frac{i}{\sqrt{b}} \operatorname{Sin}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}}-d x\right]\right)+\right. \\
& \left.\frac{1}{4 a^{3/2}} \frac{i}{\sqrt{b}}\left(-\frac{\operatorname{Cosh}[d x]}{-\frac{i}{\sqrt{a}} \sqrt{b}+b x}-\frac{1}{b} d\left(-\frac{i}{\sqrt{a}} \operatorname{CoshIntegral}\left[d\left(-\frac{i \sqrt{a}}{\sqrt{b}}+x\right)\right] \operatorname{Sin}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right]+\right.\right. \\
& \left.\left.\left.\operatorname{Cos}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}}-d x\right]\right)\right)-\frac{1}{2 a^2}\right. \\
& \left.\left(\operatorname{Cos}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}}+d x\right]-\frac{i}{\sqrt{b}} \operatorname{Sin}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}}+d x\right]\right)\right)
\end{aligned}$$

Problem 71: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Cosh}[c+d x]}{x^2 (a+b x^2)^2} d x$$

Optimal (type 4, 500 leaves, 32 steps):

$$\begin{aligned}
& -\frac{\cosh[c + dx]}{a^2 x} + \frac{\sqrt{b} \cosh[c + dx]}{4 a^2 (\sqrt{-a} - \sqrt{b} x)} - \\
& \frac{\sqrt{b} \cosh[c + dx]}{4 a^2 (\sqrt{-a} + \sqrt{b} x)} - \frac{3 \sqrt{b} \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \text{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - dx]}{4 (-a)^{5/2}} + \\
& \frac{3 \sqrt{b} \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \text{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + dx]}{4 (-a)^{5/2}} + \frac{d \text{CoshIntegral}[dx] \sinh[c]}{a^2} + \\
& \frac{d \text{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + dx] \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}]}{4 a^2} + \frac{d \text{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - dx] \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}]}{4 a^2} + \\
& \frac{d \cosh[c] \text{SinhIntegral}[dx]}{a^2} - \frac{d \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \text{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - dx]}{4 a^2} + \\
& \frac{3 \sqrt{b} \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \text{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - dx]}{4 (-a)^{5/2}} + \\
& \frac{d \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \text{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + dx]}{4 a^2} + \frac{3 \sqrt{b} \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \text{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + dx]}{4 (-a)^{5/2}}
\end{aligned}$$

Result (type 4, 675 leaves) :

$$\begin{aligned}
& \frac{1}{4 a^{5/2} x \left(a + b x^2\right)} \\
& \left(-4 a^{3/2} \cosh[c + d x] - 6 \sqrt{a} b x^2 \cosh[c + d x] + 4 a^{3/2} d x \operatorname{CoshIntegral}[d x] \sinh[c] + \right. \\
& 4 \sqrt{a} b d x^3 \operatorname{CoshIntegral}[d x] \sinh[c] + x \left(a + b x^2\right) \operatorname{CosIntegral}\left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \\
& \left. \left(-3 i \sqrt{b} \cosh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] + \sqrt{a} d \sinh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \right) + \right. \\
& x \left(a + b x^2\right) \operatorname{CosIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \left(3 i \sqrt{b} \cosh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] + \sqrt{a} d \sinh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \right) + \\
& 4 a^{3/2} d x \cosh[c] \operatorname{SinhIntegral}[d x] + 4 \sqrt{a} b d x^3 \cosh[c] \operatorname{SinhIntegral}[d x] + \\
& i a^{3/2} d x \cosh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] + \\
& i \sqrt{a} b d x^3 \cosh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] + \\
& 3 a \sqrt{b} x \sinh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] + \\
& 3 b^{3/2} x^3 \sinh\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] - \\
& i a^{3/2} d x \cosh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] - \\
& i \sqrt{a} b d x^3 \cosh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] + \\
& 3 a \sqrt{b} x \sinh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] + \\
& \left. 3 b^{3/2} x^3 \sinh\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \right)
\end{aligned}$$

Problem 72: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3 \cosh[c + d x]}{(a + b x^2)^3} dx$$

Optimal (type 4, 476 leaves, 27 steps):

$$\begin{aligned}
& -\frac{x^2 \cosh[c + d x]}{4 b (a + b x^2)^2} - \frac{\cosh[c + d x]}{4 b^2 (a + b x^2)} + \frac{d^2 \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \cosh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 b^3} + \\
& \frac{d^2 \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \cosh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 b^3} - \frac{3 d \cosh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x] \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}]}{16 \sqrt{-a} b^{5/2}} + \\
& \frac{3 d \cosh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x] \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}]}{16 \sqrt{-a} b^{5/2}} - \frac{d x \sinh[c + d x]}{8 b^2 (a + b x^2)} - \\
& \frac{3 d \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 \sqrt{-a} b^{5/2}} - \frac{d^2 \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 b^3} - \\
& \frac{3 d \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 \sqrt{-a} b^{5/2}} + \frac{d^2 \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 b^3}
\end{aligned}$$

Result (type 4, 648 leaves):

$$\begin{aligned}
& \frac{1}{16 b^2} \left(-\frac{2 \cosh[d x] (2 (a + 2 b x^2) \cosh[c] + d x (a + b x^2) \sinh[c])}{(a + b x^2)^2} - \right. \\
& \frac{2 (d x (a + b x^2) \cosh[c] + 2 (a + 2 b x^2) \sinh[c]) \sinh[d x]}{(a + b x^2)^2} + \frac{1}{\sqrt{a} \sqrt{b}} 3 i d \sinh[c] \\
& \left(\cos[\frac{\sqrt{a} d}{\sqrt{b}}] \cos\text{Integral}[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x] - \cos[\frac{\sqrt{a} d}{\sqrt{b}}] \cos\text{Integral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] + \right. \\
& \left. \sin[\frac{\sqrt{a} d}{\sqrt{b}}] \left(\sin\text{Integral}[\frac{\sqrt{a} d}{\sqrt{b}} - i d x] - \sin\text{Integral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \right) \right) - \frac{1}{b} i d^2 \sinh[c] \\
& \left(\cos\text{Integral}[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \sin[\frac{\sqrt{a} d}{\sqrt{b}}] - \cos\text{Integral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \sin[\frac{\sqrt{a} d}{\sqrt{b}}] + \right. \\
& \left. \cos[\frac{\sqrt{a} d}{\sqrt{b}}] \left(-\sin\text{Integral}[\frac{\sqrt{a} d}{\sqrt{b}} - i d x] + \sin\text{Integral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \right) \right) + \frac{1}{\sqrt{a} \sqrt{b}} \\
& 3 d \cosh[c] \left(\cos\text{Integral}[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \sin[\frac{\sqrt{a} d}{\sqrt{b}}] + \cos\text{Integral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \right. \\
& \left. \sin[\frac{\sqrt{a} d}{\sqrt{b}}] - \cos[\frac{\sqrt{a} d}{\sqrt{b}}] \left(\sin\text{Integral}[\frac{\sqrt{a} d}{\sqrt{b}} - i d x] + \sin\text{Integral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \right) \right) + \frac{1}{b} \\
& d^2 \cosh[c] \left(\cos[\frac{\sqrt{a} d}{\sqrt{b}}] \cos\text{Integral}[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x] + \cos[\frac{\sqrt{a} d}{\sqrt{b}}] \cos\text{Integral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] + \right. \\
& \left. \sin[\frac{\sqrt{a} d}{\sqrt{b}}] \left(\sin\text{Integral}[\frac{\sqrt{a} d}{\sqrt{b}} - i d x] + \sin\text{Integral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \right) \right)
\end{aligned}$$

Problem 73: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 \cosh[c + d x]}{(a + b x^2)^3} dx$$

Optimal (type 4, 746 leaves, 28 steps):

$$\begin{aligned}
& -\frac{\cosh[c + d x]}{16 a b^{3/2} (\sqrt{-a} - \sqrt{b} x)} + \frac{\cosh[c + d x]}{16 a b^{3/2} (\sqrt{-a} + \sqrt{b} x)} - \\
& \frac{x \cosh[c + d x]}{4 b (a + b x^2)^2} - \frac{\cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 (-a)^{3/2} b^{3/2}} + \\
& \frac{d^2 \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 \sqrt{-a} b^{5/2}} + \frac{\cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 (-a)^{3/2} b^{3/2}} - \\
& \frac{d^2 \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 \sqrt{-a} b^{5/2}} - \frac{d \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x] \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}]}{16 a b^2} - \\
& \frac{d \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x] \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}]}{16 a b^2} - \frac{d \sinh[c + d x]}{8 b^2 (a + b x^2)} + \\
& \frac{d \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 a b^2} + \frac{\sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 (-a)^{3/2} b^{3/2}} - \\
& \frac{d^2 \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 \sqrt{-a} b^{5/2}} - \frac{d \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 a b^2} + \\
& \frac{\sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 (-a)^{3/2} b^{3/2}} - \frac{d^2 \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 \sqrt{-a} b^{5/2}}
\end{aligned}$$

Result (type 4, 932 leaves):

$$\begin{aligned}
& \frac{1}{16 a^{3/2} b^2} \left(-\frac{2 a^{3/2} b x \cosh[c] \cosh[d x]}{(a + b x^2)^2} + \frac{2 \sqrt{a} b^2 x^3 \cosh[c] \cosh[d x]}{(a + b x^2)^2} - \right. \\
& \frac{2 a^{5/2} d \cosh[d x] \sinh[c]}{(a + b x^2)^2} - \frac{2 a^{3/2} b d x^2 \cosh[d x] \sinh[c]}{(a + b x^2)^2} + \frac{1}{\sqrt{b}} \operatorname{CosIntegral}[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \\
& \left. \left((b + a d^2) \cosh[c - \frac{i \sqrt{a} d}{\sqrt{b}}] + i \sqrt{a} \sqrt{b} d \sinh[c - \frac{i \sqrt{a} d}{\sqrt{b}}] \right) - \frac{1}{\sqrt{b}} \right. \\
& \left. i \operatorname{CosIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \left((b + a d^2) \cosh[c + \frac{i \sqrt{a} d}{\sqrt{b}}] - i \sqrt{a} \sqrt{b} d \sinh[c + \frac{i \sqrt{a} d}{\sqrt{b}}] \right) - \right. \\
& \left. \frac{2 a^{5/2} d \cosh[c] \sinh[d x]}{(a + b x^2)^2} - \frac{2 a^{3/2} b d x^2 \cosh[c] \sinh[d x]}{(a + b x^2)^2} - \frac{2 a^{3/2} b x \sinh[c] \sinh[d x]}{(a + b x^2)^2} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{2 \sqrt{a} b^2 x^3 \operatorname{Sinh}[c] \operatorname{Sinh}[d x]}{(a+b x^2)^2} - i \sqrt{a} d \cos\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{Cosh}[c] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] + \\
& i \sqrt{b} \operatorname{Cosh}[c] \sin\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] + \\
& \frac{i a d^2 \operatorname{Cosh}[c] \sin\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right]}{\sqrt{b}} - \\
& \sqrt{b} \cos\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{Sinh}[c] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] - \\
& \frac{a d^2 \cos\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{Sinh}[c] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right]}{\sqrt{b}} - \\
& \sqrt{a} d \sin\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{Sinh}[c] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] + \\
& i \sqrt{a} d \cos\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{Cosh}[c] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] - \\
& i \sqrt{b} \operatorname{Cosh}[c] \sin\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] - \\
& \frac{i a d^2 \operatorname{Cosh}[c] \sin\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right]}{\sqrt{b}} - \\
& \sqrt{b} \cos\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{Sinh}[c] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] - \\
& \frac{a d^2 \cos\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{Sinh}[c] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right]}{\sqrt{b}} - \\
& \sqrt{a} d \sin\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{Sinh}[c] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right]
\end{aligned}
\Bigg)$$

Problem 74: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x \operatorname{Cosh}[c + d x]}{(a + b x^2)^3} d x$$

Optimal (type 4, 512 leaves, 19 steps):

$$\begin{aligned}
& -\frac{\cosh[c + d x]}{4 b (a + b x^2)^2} - \frac{d^2 \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 a b^2} - \\
& \frac{d^2 \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 a b^2} + \frac{d \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x] \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}]}{16 (-a)^{3/2} b^{3/2}} - \\
& \frac{d \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x] \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}]}{16 (-a)^{3/2} b^{3/2}} - \frac{d \sinh[c + d x]}{16 a b^{3/2} (\sqrt{-a} - \sqrt{b} x)} + \frac{d \sinh[c + d x]}{16 a b^{3/2} (\sqrt{-a} + \sqrt{b} x)} + \\
& \frac{d \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 (-a)^{3/2} b^{3/2}} + \frac{d^2 \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 a b^2} + \\
& \frac{d \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 (-a)^{3/2} b^{3/2}} - \frac{d^2 \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 a b^2}
\end{aligned}$$

Result (type 4, 637 leaves) :

$$\begin{aligned}
& \frac{1}{16 a b} \left(\frac{2 \cosh[d x] (-2 a \cosh[c] + d x (a + b x^2) \sinh[c])}{(a + b x^2)^2} + \right. \\
& \frac{2 (d x (a + b x^2) \cosh[c] - 2 a \sinh[c]) \sinh[d x]}{(a + b x^2)^2} + \frac{1}{\sqrt{a} \sqrt{b}} i d \sinh[c] \\
& \left(\cos[\frac{\sqrt{a} d}{\sqrt{b}}] \operatorname{CosIntegral}[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x] - \cos[\frac{\sqrt{a} d}{\sqrt{b}}] \operatorname{CosIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] + \right. \\
& \left. \sin[\frac{\sqrt{a} d}{\sqrt{b}}] \left(\operatorname{SinIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} - i d x] - \operatorname{SinIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \right) \right) + \frac{1}{b} i d^2 \sinh[c] \\
& \left(\operatorname{CosIntegral}[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \sin[\frac{\sqrt{a} d}{\sqrt{b}}] - \operatorname{CosIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \sin[\frac{\sqrt{a} d}{\sqrt{b}}] + \right. \\
& \left. \cos[\frac{\sqrt{a} d}{\sqrt{b}}] \left(-\operatorname{SinIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} - i d x] + \operatorname{SinIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \right) \right) + \frac{1}{\sqrt{a} \sqrt{b}} \\
& d \cosh[c] \left(\operatorname{CosIntegral}[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \sin[\frac{\sqrt{a} d}{\sqrt{b}}] + \operatorname{CosIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \sin[\frac{\sqrt{a} d}{\sqrt{b}}] - \right. \\
& \left. \cos[\frac{\sqrt{a} d}{\sqrt{b}}] \left(\operatorname{SinIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} - i d x] + \operatorname{SinIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \right) \right) - \frac{1}{b} \\
& d^2 \cosh[c] \left(\cos[\frac{\sqrt{a} d}{\sqrt{b}}] \operatorname{CosIntegral}[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x] + \cos[\frac{\sqrt{a} d}{\sqrt{b}}] \operatorname{CosIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] + \right. \\
& \left. \sin[\frac{\sqrt{a} d}{\sqrt{b}}] \left(\operatorname{SinIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} - i d x] + \operatorname{SinIntegral}[\frac{\sqrt{a} d}{\sqrt{b}} + i d x] \right) \right)
\end{aligned}$$

Problem 75: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cosh[c + d x]}{(a + b x^2)^3} dx$$

Optimal (type 4, 856 leaves, 28 steps):

$$\begin{aligned}
& -\frac{\cosh[c + d x]}{16 (-a)^{3/2} \sqrt{b} (\sqrt{-a} - \sqrt{b} x)^2} - \frac{3 \cosh[c + d x]}{16 a^2 \sqrt{b} (\sqrt{-a} - \sqrt{b} x)} + \frac{\cosh[c + d x]}{16 (-a)^{3/2} \sqrt{b} (\sqrt{-a} + \sqrt{b} x)^2} + \\
& \frac{3 \cosh[c + d x]}{16 a^2 \sqrt{b} (\sqrt{-a} + \sqrt{b} x)} + \frac{3 \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 (-a)^{5/2} \sqrt{b}} + \\
& \frac{d^2 \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 (-a)^{3/2} b^{3/2}} - \frac{3 \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 (-a)^{5/2} \sqrt{b}} - \\
& \frac{d^2 \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 (-a)^{3/2} b^{3/2}} - \frac{3 d \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x] \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}]}{16 a^2 b} - \\
& \frac{3 d \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x] \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}]}{16 a^2 b} + \frac{d \sinh[c + d x]}{16 (-a)^{3/2} b (\sqrt{-a} - \sqrt{b} x)} + \\
& \frac{d \sinh[c + d x]}{16 (-a)^{3/2} b (\sqrt{-a} + \sqrt{b} x)} + \frac{3 d \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 a^2 b} - \\
& \frac{3 \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 (-a)^{5/2} \sqrt{b}} - \frac{d^2 \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 (-a)^{3/2} b^{3/2}} - \\
& \frac{3 d \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 a^2 b} - \\
& \frac{3 \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 (-a)^{5/2} \sqrt{b}} - \frac{d^2 \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 (-a)^{3/2} b^{3/2}}
\end{aligned}$$

Result (type 4, 933 leaves):

$$\begin{aligned}
& \frac{1}{16 a^2 b^{3/2}} \left(\frac{10 a b^{3/2} x \cosh[c] \cosh[d x]}{(a + b x^2)^2} + \frac{6 b^{5/2} x^3 \cosh[c] \cosh[d x]}{(a + b x^2)^2} + \right. \\
& \frac{2 a^2 \sqrt{b} d \cosh[d x] \sinh[c]}{(a + b x^2)^2} + \frac{2 a b^{3/2} d x^2 \cosh[d x] \sinh[c]}{(a + b x^2)^2} + \frac{1}{\sqrt{a}} \\
& \left. \operatorname{CosIntegral}\left[-\frac{\sqrt{a} d}{\sqrt{b}} + \frac{i}{a} d x\right] \left(i (3 b - a d^2) \cosh[c - \frac{i \sqrt{a} d}{\sqrt{b}}] - 3 \sqrt{a} \sqrt{b} d \sinh[c - \frac{i \sqrt{a} d}{\sqrt{b}}]\right)\right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{a}} i \operatorname{CosIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \\
& \left(\left(-3 b + a d^2 \right) \operatorname{Cosh}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] + 3 i \sqrt{a} \sqrt{b} d \operatorname{Sinh}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \right) + \\
& \frac{2 a^2 \sqrt{b} d \operatorname{Cosh}[c] \operatorname{Sinh}[d x]}{(a + b x^2)^2} + \frac{2 a b^{3/2} d x^2 \operatorname{Cosh}[c] \operatorname{Sinh}[d x]}{(a + b x^2)^2} + \frac{10 a b^{3/2} x \operatorname{Sinh}[c] \operatorname{Sinh}[d x]}{(a + b x^2)^2} + \\
& \frac{6 b^{5/2} x^3 \operatorname{Sinh}[c] \operatorname{Sinh}[d x]}{(a + b x^2)^2} - 3 i \sqrt{b} d \operatorname{Cos}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{Cosh}[c] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] + \\
& \frac{3 i b \operatorname{Cosh}[c] \operatorname{Sin}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right]}{\sqrt{a}} - \\
& i \sqrt{a} d^2 \operatorname{Cosh}[c] \operatorname{Sin}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] - \\
& \frac{3 b \operatorname{Cos}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{Sinh}[c] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right]}{\sqrt{a}} + \\
& \sqrt{a} d^2 \operatorname{Cos}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{Sinh}[c] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] - \\
& 3 \sqrt{b} d \operatorname{Sin}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{Sinh}[c] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] + \\
& 3 i \sqrt{b} d \operatorname{Cos}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{Cosh}[c] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] - \\
& \frac{3 i b \operatorname{Cosh}[c] \operatorname{Sin}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right]}{\sqrt{a}} + \\
& i \sqrt{a} d^2 \operatorname{Cosh}[c] \operatorname{Sin}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] - \\
& \frac{3 b \operatorname{Cos}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{Sinh}[c] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right]}{\sqrt{a}} + \\
& \sqrt{a} d^2 \operatorname{Cos}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{Sinh}[c] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] - \\
& 3 \sqrt{b} d \operatorname{Sin}\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \operatorname{Sinh}[c] \operatorname{SinIntegral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right]
\end{aligned}$$

Problem 76: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\cosh[c + d x]}{x (a + b x^2)^3} dx$$

Optimal (type 4, 730 leaves, 41 steps):

$$\begin{aligned} & \frac{\cosh[c + d x]}{4 a (a + b x^2)^2} + \frac{\cosh[c + d x]}{2 a^2 (a + b x^2)} + \frac{\cosh[c] \cosh\text{Integral}[d x]}{a^3} - \\ & \frac{\cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \cosh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{2 a^3} + \frac{d^2 \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \cosh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 a^2 b} - \\ & \frac{\cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \cosh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{2 a^3} + \frac{d^2 \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \cosh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 a^2 b} + \\ & \frac{5 d \cosh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x] \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}]}{16 (-a)^{5/2} \sqrt{b}} - \frac{5 d \cosh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x] \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}]}{16 (-a)^{5/2} \sqrt{b}} + \\ & \frac{d \sinh[c + d x]}{16 a^2 \sqrt{b} (\sqrt{-a} - \sqrt{b} x)} - \frac{d \sinh[c + d x]}{16 a^2 \sqrt{b} (\sqrt{-a} + \sqrt{b} x)} + \frac{\sinh[c] \sinh\text{Integral}[d x]}{a^3} + \\ & \frac{5 d \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 (-a)^{5/2} \sqrt{b}} + \frac{\sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{2 a^3} - \\ & \frac{d^2 \sinh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 a^2 b} + \frac{5 d \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 (-a)^{5/2} \sqrt{b}} - \\ & \frac{\sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{2 a^3} + \frac{d^2 \sinh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \sinh\text{Integral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 a^2 b} \end{aligned}$$

Result (type 4, 1558 leaves):

$$\begin{aligned} & \frac{1}{16 a^3 b (a + b x^2)^2} \\ & \left(12 a^2 b \cosh[c + d x] + 8 a b^2 x^2 \cosh[c + d x] + 16 b (a + b x^2)^2 \cosh[c] \cosh\text{Integral}[d x] - \right. \\ & 8 a^2 b \cosh[c - \frac{i \sqrt{a} d}{\sqrt{b}}] \cosh\text{Integral}[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)] + \\ & a^3 d^2 \cosh[c - \frac{i \sqrt{a} d}{\sqrt{b}}] \cosh\text{Integral}[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)] - \\ & 16 a b^2 x^2 \cosh[c - \frac{i \sqrt{a} d}{\sqrt{b}}] \cosh\text{Integral}[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)] + 2 a^2 b d^2 x^2 \cosh[c - \frac{i \sqrt{a} d}{\sqrt{b}}] \\ & \cosh\text{Integral}[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)] - 8 b^3 x^4 \cosh[c - \frac{i \sqrt{a} d}{\sqrt{b}}] \cosh\text{Integral}[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)] + \\ & \left. a b^2 d^2 x^4 \cosh[c - \frac{i \sqrt{a} d}{\sqrt{b}}] \cosh\text{Integral}[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x \right)] \right) \end{aligned}$$

$$\begin{aligned}
& 5 i a^{5/2} \sqrt{b} d \operatorname{CoshIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \operatorname{Sinh}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] - \\
& 10 i a^{3/2} b^{3/2} d x^2 \operatorname{CoshIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \operatorname{Sinh}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] - \\
& 5 i \sqrt{a} b^{5/2} d x^4 \operatorname{CoshIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \operatorname{Sinh}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] + \\
& (a + b x^2)^2 \operatorname{CoshIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \\
& \left((-8 b + a d^2) \operatorname{Cosh}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] + 5 i \sqrt{a} \sqrt{b} d \operatorname{Sinh}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \right) - \\
& 2 a^2 b d x \operatorname{Sinh}\left[c + d x\right] - 2 a b^2 d x^3 \operatorname{Sinh}\left[c + d x\right] + 16 a^2 b \operatorname{Sinh}[c] \operatorname{SinhIntegral}[d x] + \\
& 32 a b^2 x^2 \operatorname{Sinh}[c] \operatorname{SinhIntegral}[d x] + 16 b^3 x^4 \operatorname{Sinh}[c] \operatorname{SinhIntegral}[d x] - \\
& 5 i a^{5/2} \sqrt{b} d \operatorname{Cosh}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] - \\
& 10 i a^{3/2} b^{3/2} d x^2 \operatorname{Cosh}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] - \\
& 5 i \sqrt{a} b^{5/2} d x^4 \operatorname{Cosh}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] - \\
& 8 a^2 b \operatorname{Sinh}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + \\
& a^3 d^2 \operatorname{Sinh}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] - \\
& 16 a b^2 x^2 \operatorname{Sinh}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + \\
& 2 a^2 b d^2 x^2 \operatorname{Sinh}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] - \\
& 8 b^3 x^4 \operatorname{Sinh}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] + \\
& a b^2 d^2 x^4 \operatorname{Sinh}\left[c - \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] - \\
& 5 i a^{5/2} \sqrt{b} d \operatorname{Cosh}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] - \\
& 10 i a^{3/2} b^{3/2} d x^2 \operatorname{Cosh}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] - \\
& 5 i \sqrt{a} b^{5/2} d x^4 \operatorname{Cosh}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] +
\end{aligned}$$

$$\begin{aligned}
& 8 a^2 b \operatorname{Sinh}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] - \\
& a^3 d^2 \operatorname{Sinh}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] + \\
& 16 a b^2 x^2 \operatorname{Sinh}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] - \\
& 2 a^2 b d^2 x^2 \operatorname{Sinh}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] + \\
& 8 b^3 x^4 \operatorname{Sinh}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] - \\
& a b^2 d^2 x^4 \operatorname{Sinh}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right]
\end{aligned}$$

Problem 77: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Cosh}[c + d x]}{x^2 (a + b x^2)^3} d x$$

Optimal (type 4, 874 leaves, 60 steps):

$$\begin{aligned}
& -\frac{\cosh[c+d x]}{a^3 x} - \frac{\sqrt{b} \cosh[c+d x]}{16 (-a)^{5/2} (\sqrt{-a} - \sqrt{b} x)^2} + \frac{7 \sqrt{b} \cosh[c+d x]}{16 a^3 (\sqrt{-a} - \sqrt{b} x)} + \frac{\sqrt{b} \cosh[c+d x]}{16 (-a)^{5/2} (\sqrt{-a} + \sqrt{b} x)^2} - \\
& \frac{7 \sqrt{b} \cosh[c+d x]}{16 a^3 (\sqrt{-a} + \sqrt{b} x)} + \frac{15 \sqrt{b} \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 (-a)^{7/2}} + \\
& \frac{d^2 \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 (-a)^{5/2} \sqrt{b}} - \frac{15 \sqrt{b} \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 (-a)^{7/2}} - \\
& \frac{d^2 \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 (-a)^{5/2} \sqrt{b}} + \frac{d \operatorname{CoshIntegral}[d x] \operatorname{Sinh}[c]}{a^3} + \\
& \frac{7 d \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x] \operatorname{Sinh}[c - \frac{\sqrt{-a} d}{\sqrt{b}}]}{16 a^3} + \frac{7 d \operatorname{CoshIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x] \operatorname{Sinh}[c + \frac{\sqrt{-a} d}{\sqrt{b}}]}{16 a^3} + \\
& \frac{d \operatorname{Sinh}[c + d x]}{16 (-a)^{5/2} (\sqrt{-a} - \sqrt{b} x)} + \frac{d \operatorname{Sinh}[c + d x]}{16 (-a)^{5/2} (\sqrt{-a} + \sqrt{b} x)} + \\
& \frac{d \cosh[c] \operatorname{SinhIntegral}[d x]}{a^3} - \frac{7 d \cosh[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 a^3} - \\
& \frac{15 \sqrt{b} \operatorname{Sinh}[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 (-a)^{7/2}} - \\
& \frac{d^2 \operatorname{Sinh}[c + \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} - d x]}{16 (-a)^{5/2} \sqrt{b}} + \frac{7 d \cosh[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 a^3} - \\
& \frac{15 \sqrt{b} \operatorname{Sinh}[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 (-a)^{7/2}} - \frac{d^2 \operatorname{Sinh}[c - \frac{\sqrt{-a} d}{\sqrt{b}}] \operatorname{SinhIntegral}[\frac{\sqrt{-a} d}{\sqrt{b}} + d x]}{16 (-a)^{5/2} \sqrt{b}}
\end{aligned}$$

Result (type 4, 1359 leaves) :

$$\begin{aligned}
& \frac{1}{16 a^{7/2} \sqrt{b} x (a + b x^2)^2} \\
& \left(-16 a^{5/2} \sqrt{b} \cosh[c + d x] - 50 a^{3/2} b^{3/2} x^2 \cosh[c + d x] - 30 \sqrt{a} b^{5/2} x^4 \cosh[c + d x] + \right. \\
& 16 a^{5/2} \sqrt{b} d x \operatorname{CoshIntegral}[d x] \operatorname{Sinh}[c] + 32 a^{3/2} b^{3/2} d x^3 \operatorname{CoshIntegral}[d x] \operatorname{Sinh}[c] + \\
& 16 \sqrt{a} b^{5/2} d x^5 \operatorname{CoshIntegral}[d x] \operatorname{Sinh}[c] + x (a + b x^2)^2 \operatorname{CoshIntegral}\left[d \left(\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \\
& \left. \left(-i (15 b - a d^2) \cosh[c - \frac{i \sqrt{a} d}{\sqrt{b}}] + 7 \sqrt{a} \sqrt{b} d \operatorname{Sinh}[c - \frac{i \sqrt{a} d}{\sqrt{b}}] \right) + \right. \\
& \left. x (a + b x^2)^2 \operatorname{CoshIntegral}\left[d \left(-\frac{i \sqrt{a}}{\sqrt{b}} + x\right)\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{\text{i} (15 b - a d^2) \cosh[c + \frac{\text{i} \sqrt{a} d}{\sqrt{b}}] + 7 \sqrt{a} \sqrt{b} d \sinh[c + \frac{\text{i} \sqrt{a} d}{\sqrt{b}}]}{\sqrt{b}} \right) - \\
& 2 a^{5/2} \sqrt{b} d x \sinh[c + d x] - 2 a^{3/2} b^{3/2} d x^3 \sinh[c + d x] + \\
& 16 a^{5/2} \sqrt{b} d x \cosh[c] \sinh\text{Integral}[d x] + 32 a^{3/2} b^{3/2} d x^3 \cosh[c] \sinh\text{Integral}[d x] + \\
& 16 \sqrt{a} b^{5/2} d x^5 \cosh[c] \sinh\text{Integral}[d x] + \\
& 7 a^{5/2} \sqrt{b} d x \cosh[c - \frac{\text{i} \sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}\left[d \left(\frac{\text{i} \sqrt{a}}{\sqrt{b}} + x\right)\right] + \\
& 14 a^{3/2} b^{3/2} d x^3 \cosh[c - \frac{\text{i} \sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}\left[d \left(\frac{\text{i} \sqrt{a}}{\sqrt{b}} + x\right)\right] + \\
& 7 \sqrt{a} b^{5/2} d x^5 \cosh[c - \frac{\text{i} \sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}\left[d \left(\frac{\text{i} \sqrt{a}}{\sqrt{b}} + x\right)\right] - \\
& 15 \text{i} a^2 b x \sinh[c - \frac{\text{i} \sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}\left[d \left(\frac{\text{i} \sqrt{a}}{\sqrt{b}} + x\right)\right] + \\
& \text{i} a^3 d^2 x \sinh[c - \frac{\text{i} \sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}\left[d \left(\frac{\text{i} \sqrt{a}}{\sqrt{b}} + x\right)\right] - \\
& 30 \text{i} a b^2 x^3 \sinh[c - \frac{\text{i} \sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}\left[d \left(\frac{\text{i} \sqrt{a}}{\sqrt{b}} + x\right)\right] + \\
& 2 \text{i} a^2 b d^2 x^3 \sinh[c - \frac{\text{i} \sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}\left[d \left(\frac{\text{i} \sqrt{a}}{\sqrt{b}} + x\right)\right] - \\
& 15 \text{i} b^3 x^5 \sinh[c - \frac{\text{i} \sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}\left[d \left(\frac{\text{i} \sqrt{a}}{\sqrt{b}} + x\right)\right] + \\
& \text{i} a b^2 d^2 x^5 \sinh[c - \frac{\text{i} \sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}\left[d \left(\frac{\text{i} \sqrt{a}}{\sqrt{b}} + x\right)\right] - \\
& 7 a^{5/2} \sqrt{b} d x \cosh[c + \frac{\text{i} \sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}\left[\frac{\text{i} \sqrt{a} d}{\sqrt{b}} - d x\right] - \\
& 14 a^{3/2} b^{3/2} d x^3 \cosh[c + \frac{\text{i} \sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}\left[\frac{\text{i} \sqrt{a} d}{\sqrt{b}} - d x\right] - \\
& 7 \sqrt{a} b^{5/2} d x^5 \cosh[c + \frac{\text{i} \sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}\left[\frac{\text{i} \sqrt{a} d}{\sqrt{b}} - d x\right] - \\
& 15 \text{i} a^2 b x \sinh[c + \frac{\text{i} \sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}\left[\frac{\text{i} \sqrt{a} d}{\sqrt{b}} - d x\right] + \\
& \text{i} a^3 d^2 x \sinh[c + \frac{\text{i} \sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}\left[\frac{\text{i} \sqrt{a} d}{\sqrt{b}} - d x\right] - \\
& 30 \text{i} a b^2 x^3 \sinh[c + \frac{\text{i} \sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}\left[\frac{\text{i} \sqrt{a} d}{\sqrt{b}} - d x\right] + \\
& 2 \text{i} a^2 b d^2 x^3 \sinh[c + \frac{\text{i} \sqrt{a} d}{\sqrt{b}}] \sinh\text{Integral}\left[\frac{\text{i} \sqrt{a} d}{\sqrt{b}} - d x\right] -
\end{aligned}$$

$$15 i b^3 x^5 \operatorname{Sinh}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right] + \\ i a b^2 d^2 x^5 \operatorname{Sinh}\left[c + \frac{i \sqrt{a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{i \sqrt{a} d}{\sqrt{b}} - d x\right]$$

Problem 78: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Cosh}[c + d x]}{x^3 (a + b x^2)^3} dx$$

Optimal (type 4, 791 leaves, 46 steps):

$$-\frac{\operatorname{Cosh}[c + d x]}{2 a^3 x^2} - \frac{b \operatorname{Cosh}[c + d x]}{4 a^2 (a + b x^2)^2} - \frac{b \operatorname{Cosh}[c + d x]}{a^3 (a + b x^2)} - \\ \frac{3 b \operatorname{Cosh}[c] \operatorname{CoshIntegral}[d x]}{a^4} + \frac{d^2 \operatorname{Cosh}[c] \operatorname{CoshIntegral}[d x]}{2 a^3} + \\ \frac{3 b \operatorname{Cosh}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{2 a^4} - \frac{d^2 \operatorname{Cosh}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{16 a^3} + \\ \frac{3 b \operatorname{Cosh}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{2 a^4} - \frac{d^2 \operatorname{Cosh}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{16 a^3} + \\ \frac{9 \sqrt{b} d \operatorname{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right] \operatorname{Sinh}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 (-a)^{7/2}} - \\ \frac{9 \sqrt{b} d \operatorname{CoshIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right] \operatorname{Sinh}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right]}{16 (-a)^{7/2}} - \frac{d \operatorname{Sinh}[c + d x]}{2 a^3 x} - \\ \frac{\sqrt{b} d \operatorname{Sinh}[c + d x]}{16 a^3 (\sqrt{-a} - \sqrt{b} x)} + \frac{\sqrt{b} d \operatorname{Sinh}[c + d x]}{16 a^3 (\sqrt{-a} + \sqrt{b} x)} - \frac{3 b \operatorname{Sinh}[c] \operatorname{SinhIntegral}[d x]}{a^4} + \\ \frac{d^2 \operatorname{Sinh}[c] \operatorname{SinhIntegral}[d x]}{2 a^3} + \frac{9 \sqrt{b} d \operatorname{Cosh}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{16 (-a)^{7/2}} - \\ \frac{3 b \operatorname{Sinh}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{2 a^4} + \frac{d^2 \operatorname{Sinh}\left[c + \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} - d x\right]}{16 a^3} + \\ \frac{9 \sqrt{b} d \operatorname{Cosh}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{16 (-a)^{7/2}} + \\ \frac{3 b \operatorname{Sinh}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{2 a^4} - \frac{d^2 \operatorname{Sinh}\left[c - \frac{\sqrt{-a} d}{\sqrt{b}}\right] \operatorname{SinhIntegral}\left[\frac{\sqrt{-a} d}{\sqrt{b}} + d x\right]}{16 a^3}$$

Result (type 4, 998 leaves):

$$\begin{aligned}
& -\frac{1}{16 a^4} \left(\frac{1}{x^2 (a + b x^2)^2} 2 a \cosh[d x] \right. \\
& \quad \left(2 (2 a^2 + 9 a b x^2 + 6 b^2 x^4) \cosh[c] + d x (4 a^2 + 7 a b x^2 + 3 b^2 x^4) \sinh[c] \right) + \frac{1}{x^2 (a + b x^2)^2} \\
& \quad 2 a (d x (4 a^2 + 7 a b x^2 + 3 b^2 x^4) \cosh[c] + 2 (2 a^2 + 9 a b x^2 + 6 b^2 x^4) \sinh[c]) \sinh[d x] + \\
& \quad 8 (6 b - a d^2) (\cosh[c] \cosh\text{Integral}[d x] + \sinh[c] \sinh\text{Integral}[d x]) - 9 i \sqrt{a} \sqrt{b} d \\
& \quad \sinh[c] \left(\cos\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \cos\text{Integral}\left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] - \cos\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \cos\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] + \right. \\
& \quad \left. \sin\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \left(\sin\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] - \sin\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \right) \right) + 24 i b \sinh[c] \\
& \quad \left(\cos\text{Integral}\left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \sin\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] - \cos\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \sin\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] + \right. \\
& \quad \left. \cos\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \left(-\sin\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] + \sin\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \right) \right) - i a d^2 \sinh[c] \\
& \quad \left(\cos\text{Integral}\left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \sin\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] - \cos\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \sin\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] + \right. \\
& \quad \left. \cos\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \left(-\sin\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] + \sin\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \right) \right) - 9 \sqrt{a} \sqrt{b} d \\
& \quad \cosh[c] \left(\cos\text{Integral}\left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \sin\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] + \cos\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \sin\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] - \right. \\
& \quad \left. \cos\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \left(\sin\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] + \sin\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \right) \right) - 24 b \cosh[c] \\
& \quad \left(\cos\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \cos\text{Integral}\left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] + \cos\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \cos\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] + \right. \\
& \quad \left. \sin\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \left(\sin\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] + \sin\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \right) \right) + a d^2 \cosh[c] \\
& \quad \left(\cos\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \cos\text{Integral}\left[-\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] + \cos\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \cos\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] + \right. \\
& \quad \left. \sin\left[\frac{\sqrt{a} d}{\sqrt{b}}\right] \left(\sin\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} - i d x\right] + \sin\text{Integral}\left[\frac{\sqrt{a} d}{\sqrt{b}} + i d x\right] \right) \right)
\end{aligned}$$

Problem 94: Result is not expressed in closed-form.

$$\int \frac{x^4 \cosh[c + d x]}{a + b x^3} dx$$

Optimal (type 4, 373 leaves, 15 steps):

$$\begin{aligned}
& -\frac{\cosh[c + d x]}{b d^2} + \frac{(-1)^{2/3} a^{2/3} \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \text{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{3 b^{5/3}} - \\
& \frac{(-1)^{1/3} a^{2/3} \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \text{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x]}{3 b^{5/3}} + \\
& \frac{a^{2/3} \cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \text{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{3 b^{5/3}} + \frac{x \sinh[c + d x]}{b d} - \\
& \frac{(-1)^{2/3} a^{2/3} \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \text{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{3 b^{5/3}} + \\
& \frac{a^{2/3} \sinh[c - \frac{a^{1/3} d}{b^{1/3}}] \text{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{3 b^{5/3}} - \\
& \frac{(-1)^{1/3} a^{2/3} \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \text{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x]}{3 b^{5/3}}
\end{aligned}$$

Result (type 7, 213 leaves):

$$\begin{aligned}
& -\frac{1}{6 b^2 d^2} \left(a d^2 \text{RootSum}[a + b \#1^3 \&, \right. \\
& \left. \frac{1}{\#1} (\cosh[c + d \#1] \text{CoshIntegral}[d (x - \#1)] - \text{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] - \right. \\
& \left. \cosh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] + \sinh[c + d \#1] \text{SinhIntegral}[d (x - \#1)]) \&] + \right. \\
& a d^2 \text{RootSum}[a + b \#1^3 \&, \frac{1}{\#1} (\cosh[c + d \#1] \text{CoshIntegral}[d (x - \#1)] + \\
& \left. \cosh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \sinh[c + d \#1] + \cosh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] + \right. \\
& \left. \sinh[c + d \#1] \text{SinhIntegral}[d (x - \#1)]) \&] + 6 b (\cosh[c + d x] - d x \sinh[c + d x]) \right)
\end{aligned}$$

Problem 95: Result is not expressed in closed-form.

$$\int \frac{x^3 \cosh[c + d x]}{a + b x^3} dx$$

Optimal (type 4, 358 leaves, 14 steps):

$$\begin{aligned}
& \frac{(-1)^{1/3} a^{1/3} \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx]}{3 b^{4/3}} - \\
& \frac{(-1)^{2/3} a^{1/3} \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - dx]}{3 b^{4/3}} - \\
& \frac{a^{1/3} \cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + dx]}{3 b^{4/3}} + \frac{\sinh[c + dx]}{b d} - \\
& \frac{(-1)^{1/3} a^{1/3} \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx]}{3 b^{4/3}} - \\
& \frac{a^{1/3} \sinh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + dx]}{3 b^{4/3}} - \\
& \frac{(-1)^{2/3} a^{1/3} \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx]}{3 b^{4/3}}
\end{aligned}$$

Result (type 7, 198 leaves):

$$\begin{aligned}
& -\frac{1}{6 b^2 d} \left(a d \operatorname{RootSum}[a + b \#1^3 \&, \right. \\
& \frac{1}{\#1^2} (\cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] - \operatorname{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] - \\
& \cosh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] + \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)]) \&] + \\
& a d \operatorname{RootSum}[a + b \#1^3 \&, \frac{1}{\#1^2} (\cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] + \\
& \operatorname{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] + \cosh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] + \\
& \left. \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] \&] - 6 b \sinh[c + dx] \right)
\end{aligned}$$

Problem 96: Result is not expressed in closed-form.

$$\int \frac{x^2 \cosh[c + dx]}{a + b x^3} dx$$

Optimal (type 4, 283 leaves, 11 steps):

$$\begin{aligned}
& \frac{\cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx]}{3 b} + \\
& \frac{\cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - dx]}{3 b} + \\
& \frac{\cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + dx]}{3 b} - \frac{\sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx]}{3 b} + \\
& \frac{\sinh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + dx]}{3 b} + \frac{\sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx]}{3 b}
\end{aligned}$$

Result (type 7, 170 leaves) :

$$\frac{1}{6 b} \left(\text{RootSum}[a + b \#1^3 \&, \right.$$

$$\text{Cosh}[c + d \#1] \text{CoshIntegral}[d (x - \#1)] - \text{CoshIntegral}[d (x - \#1)] \text{Sinh}[c + d \#1] -$$

$$\text{Cosh}[c + d \#1] \text{SinhIntegral}[d (x - \#1)] + \text{Sinh}[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \&] +$$

$$\text{RootSum}[a + b \#1^3 \&, \text{Cosh}[c + d \#1] \text{CoshIntegral}[d (x - \#1)] +$$

$$\text{CoshIntegral}[d (x - \#1)] \text{Sinh}[c + d \#1] + \text{Cosh}[c + d \#1] \text{SinhIntegral}[d (x - \#1)] +$$

$$\text{Sinh}[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \&])$$

Problem 97: Result is not expressed in closed-form.

$$\int \frac{x \cosh[c + d x]}{a + b x^3} dx$$

Optimal (type 4, 345 leaves, 11 steps) :

$$-\frac{(-1)^{2/3} \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \text{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{3 a^{1/3} b^{2/3}} +$$

$$\frac{(-1)^{1/3} \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \text{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x]}{3 a^{1/3} b^{2/3}} -$$

$$\frac{\cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \text{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{3 a^{1/3} b^{2/3}} +$$

$$\frac{(-1)^{2/3} \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \text{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{3 a^{1/3} b^{2/3}} -$$

$$\frac{\sinh[c - \frac{a^{1/3} d}{b^{1/3}}] \text{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{3 a^{1/3} b^{2/3}} +$$

$$\frac{(-1)^{1/3} \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \text{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x]}{3 a^{1/3} b^{2/3}}$$

Result (type 7, 180 leaves) :

$$\frac{1}{6 b} \left(\text{RootSum}[a + b \#1^3 \&, \right.$$

$$\frac{1}{\#1} (\text{Cosh}[c + d \#1] \text{CoshIntegral}[d (x - \#1)] - \text{CoshIntegral}[d (x - \#1)] \text{Sinh}[c + d \#1] -$$

$$\text{Cosh}[c + d \#1] \text{SinhIntegral}[d (x - \#1)] + \text{Sinh}[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \&] +$$

$$\text{RootSum}[a + b \#1^3 \&, \frac{1}{\#1} (\text{Cosh}[c + d \#1] \text{CoshIntegral}[d (x - \#1)] +$$

$$\text{CoshIntegral}[d (x - \#1)] \text{Sinh}[c + d \#1] + \text{Cosh}[c + d \#1] \text{SinhIntegral}[d (x - \#1)] +$$

$$\text{Sinh}[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \&])$$

Problem 98: Result is not expressed in closed-form.

$$\int \frac{\cosh[c + d x]}{a + b x^3} dx$$

Optimal (type 4, 345 leaves, 11 steps):

$$\begin{aligned} & -\frac{(-1)^{1/3} \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{3 a^{2/3} b^{1/3}} + \\ & \frac{(-1)^{2/3} \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x\right]}{3 a^{2/3} b^{1/3}} + \\ & \frac{\cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{3 a^{2/3} b^{1/3}} + \\ & \frac{(-1)^{1/3} \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right]}{3 a^{2/3} b^{1/3}} + \\ & \frac{\sinh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right]}{3 a^{2/3} b^{1/3}} + \\ & \frac{(-1)^{2/3} \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right]}{3 a^{2/3} b^{1/3}} \end{aligned}$$

Result (type 7, 180 leaves):

$$\begin{aligned} & \frac{1}{6 b} \left(\operatorname{RootSum}[a + b \#1^3 \&, \right. \\ & \frac{1}{\#1^2} (\cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] - \operatorname{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] - \\ & \quad \cosh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] + \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)]) \&] + \\ & \operatorname{RootSum}[a + b \#1^3 \&, \frac{1}{\#1^2} (\cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] + \\ & \quad \operatorname{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] + \cosh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] + \\ & \quad \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)]) \&] \left. \right) \end{aligned}$$

Problem 99: Result is not expressed in closed-form.

$$\int \frac{\cosh[c + d x]}{x (a + b x^3)} dx$$

Optimal (type 4, 303 leaves, 16 steps):

$$\begin{aligned}
& \frac{\cosh[c] \cosh\text{Integral}[dx]}{a} - \frac{\cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \cosh\text{Integral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]}{3 a} - \\
& \frac{\cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \cosh\text{Integral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - dx\right]}{3 a} - \frac{\cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \cosh\text{Integral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right]}{3 a} + \\
& \frac{\sinh[c] \sinh\text{Integral}[dx]}{a} + \frac{\sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \sinh\text{Integral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]}{3 a} - \\
& \frac{\sinh[c - \frac{a^{1/3} d}{b^{1/3}}] \sinh\text{Integral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right]}{3 a} - \frac{\sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \sinh\text{Integral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right]}{3 a}
\end{aligned}$$

Result (type 7, 186 leaves):

$$\begin{aligned}
& -\frac{1}{6 a} (-6 \cosh[c] \cosh\text{Integral}[dx] + \text{RootSum}[a + b \#1^3 \&, \\
& \cosh[c + d \#1] \cosh\text{Integral}[d (x - \#1)] - \cosh\text{Integral}[d (x - \#1)] \sinh[c + d \#1] - \\
& \cosh[c + d \#1] \sinh\text{Integral}[d (x - \#1)] + \sinh[c + d \#1] \sinh\text{Integral}[d (x - \#1)] \&] + \\
& \text{RootSum}[a + b \#1^3 \&, \cosh[c + d \#1] \cosh\text{Integral}[d (x - \#1)] + \\
& \cosh\text{Integral}[d (x - \#1)] \sinh[c + d \#1] + \cosh[c + d \#1] \sinh\text{Integral}[d (x - \#1)] + \\
& \sinh[c + d \#1] \sinh\text{Integral}[d (x - \#1)] \&] - 6 \sinh[c] \sinh\text{Integral}[dx])
\end{aligned}$$

Problem 100: Result is not expressed in closed-form.

$$\int \frac{\cosh[c + dx]}{x^2 (a + b x^3)} dx$$

Optimal (type 4, 381 leaves, 17 steps):

$$\begin{aligned}
& -\frac{\cosh[c + dx]}{a x} + \frac{(-1)^{2/3} b^{1/3} \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \cosh\text{Integral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]}{3 a^{4/3}} - \\
& \frac{(-1)^{1/3} b^{1/3} \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \cosh\text{Integral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - dx\right]}{3 a^{4/3}} + \\
& \frac{b^{1/3} \cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \cosh\text{Integral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right]}{3 a^{4/3}} + \frac{d \cosh\text{Integral}[dx] \sinh[c]}{a} + \\
& \frac{d \cosh[c] \sinh\text{Integral}[dx]}{a} - \frac{(-1)^{2/3} b^{1/3} \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \sinh\text{Integral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]}{3 a^{4/3}} + \\
& \frac{b^{1/3} \sinh[c - \frac{a^{1/3} d}{b^{1/3}}] \sinh\text{Integral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right]}{3 a^{4/3}} - \\
& \frac{(-1)^{1/3} b^{1/3} \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \sinh\text{Integral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right]}{3 a^{4/3}}
\end{aligned}$$

Result (type 7, 215 leaves):

$$\begin{aligned}
& -\frac{1}{6 a x} \left(6 \cosh[c + d x] + x \operatorname{RootSum}[a + b \#^3 \&, \right. \\
& \quad \left. \frac{1}{\#^1} (\cosh[c + d \#^1] \operatorname{CoshIntegral}[d (x - \#^1)] - \operatorname{CoshIntegral}[d (x - \#^1)] \sinh[c + d \#^1] - \right. \\
& \quad \left. \cosh[c + d \#^1] \operatorname{SinhIntegral}[d (x - \#^1)] + \sinh[c + d \#^1] \operatorname{SinhIntegral}[d (x - \#^1)]) \& + \right. \\
& \quad \left. x \operatorname{RootSum}[a + b \#^3 \&, \frac{1}{\#^1} (\cosh[c + d \#^1] \operatorname{CoshIntegral}[d (x - \#^1)] + \right. \\
& \quad \left. \operatorname{CoshIntegral}[d (x - \#^1)] \sinh[c + d \#^1] + \cosh[c + d \#^1] \operatorname{SinhIntegral}[d (x - \#^1)] + \right. \\
& \quad \left. \sinh[c + d \#^1] \operatorname{SinhIntegral}[d (x - \#^1)]) \& - \right. \\
& \quad \left. 6 d x \operatorname{CoshIntegral}[d x] \sinh[c] - 6 d x \cosh[c] \operatorname{SinhIntegral}[d x] \right)
\end{aligned}$$

Problem 101: Result is not expressed in closed-form.

$$\int \frac{\cosh[c + d x]}{x^3 (a + b x^3)} dx$$

Optimal (type 4, 410 leaves, 18 steps):

$$\begin{aligned}
& -\frac{\cosh[c + d x]}{2 a x^2} + \frac{d^2 \cosh[c] \operatorname{CoshIntegral}[d x]}{2 a} + \\
& \frac{(-1)^{1/3} b^{2/3} \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{3 a^{5/3}} - \\
& \frac{(-1)^{2/3} b^{2/3} \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x]}{3 a^{5/3}} - \\
& \frac{b^{2/3} \cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{3 a^{5/3}} - \frac{d \sinh[c + d x]}{2 a x} + \frac{d^2 \sinh[c] \operatorname{SinhIntegral}[d x]}{2 a} - \\
& \frac{(-1)^{1/3} b^{2/3} \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{3 a^{5/3}} - \\
& \frac{b^{2/3} \sinh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{3 a^{5/3}} - \\
& \frac{(-1)^{2/3} b^{2/3} \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x]}{3 a^{5/3}}
\end{aligned}$$

Result (type 7, 237 leaves):

$$\begin{aligned}
& -\frac{1}{6 a x^2} \left(3 \cosh[c + d x] - 3 d^2 x^2 \cosh[c] \operatorname{CoshIntegral}[d x] + \right. \\
& \quad x^2 \operatorname{RootSum}[a + b \#1^3 \&, \frac{1}{\#1^2} (\cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] - \\
& \quad \cosh[c + d \#1] \sinh[c + d \#1] - \cosh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] + \\
& \quad \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)]) \&] + x^2 \operatorname{RootSum}[a + b \#1^3 \&, \\
& \quad \frac{1}{\#1^2} (\cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] + \cosh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] \sinh[c + d \#1] + \\
& \quad \cosh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] + \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)]) \&] + \\
& \quad \left. 3 d x \sinh[c + d x] - 3 d^2 x^2 \sinh[c] \operatorname{SinhIntegral}[d x] \right)
\end{aligned}$$

Problem 102: Result is not expressed in closed-form.

$$\int \frac{x^3 \cosh[c + d x]}{(a + b x^3)^2} dx$$

Optimal (type 4, 718 leaves, 23 steps):

$$\begin{aligned}
& -\frac{x \cosh[c + d x]}{3 b (a + b x^3)} - \frac{(-1)^{1/3} \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{9 a^{2/3} b^{4/3}} + \\
& \frac{(-1)^{2/3} \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x]}{9 a^{2/3} b^{4/3}} + \\
& \frac{\cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{9 a^{2/3} b^{4/3}} - \frac{d \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x] \sinh[c - \frac{a^{1/3} d}{b^{1/3}}]}{9 a^{1/3} b^{5/3}} - \\
& \frac{(-1)^{2/3} d \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x] \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}]}{9 a^{1/3} b^{5/3}} + \\
& \frac{(-1)^{1/3} d \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x] \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}]}{9 a^{1/3} b^{5/3}} + \\
& \frac{(-1)^{2/3} d \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{9 a^{1/3} b^{5/3}} + \\
& \frac{(-1)^{1/3} \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{9 a^{2/3} b^{4/3}} - \\
& \frac{d \cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{9 a^{1/3} b^{5/3}} + \frac{\sinh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{9 a^{2/3} b^{4/3}} + \\
& \frac{(-1)^{1/3} d \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x]}{9 a^{1/3} b^{5/3}} + \\
& \frac{(-1)^{2/3} \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x]}{9 a^{2/3} b^{4/3}}
\end{aligned}$$

Result (type 7, 363 leaves) :

$$\begin{aligned} & \frac{1}{18 b^2} \left(-\frac{6 b x \cosh[c + d x]}{a + b x^3} - \text{RootSum}[a + b \#1^3 \&, \right. \\ & \quad \left. \frac{1}{\#1^2} (-\cosh[c + d \#1] \cosh[\text{Integral}[d (x - \#1)] + \cosh[\text{Integral}[d (x - \#1)] \sinh[c + d \#1] + \right. \\ & \quad \left. \cosh[c + d \#1] \sinh[\text{Integral}[d (x - \#1)] - \sinh[c + d \#1] \sinh[\text{Integral}[d (x - \#1)] + \right. \\ & \quad \left. d \cosh[c + d \#1] \cosh[\text{Integral}[d (x - \#1)] \#1 - \right. \\ & \quad \left. d \cosh[\text{Integral}[d (x - \#1)] \sinh[c + d \#1] \#1 - d \cosh[c + d \#1] \right. \\ & \quad \left. \sinh[\text{Integral}[d (x - \#1)] \#1 + d \sinh[c + d \#1] \sinh[\text{Integral}[d (x - \#1)] \#1] \&] + \right. \\ & \quad \left. \text{RootSum}[a + b \#1^3 \&, \frac{1}{\#1^2} (\cosh[c + d \#1] \cosh[\text{Integral}[d (x - \#1)] + \right. \\ & \quad \left. \cosh[\text{Integral}[d (x - \#1)] \sinh[c + d \#1] + \cosh[c + d \#1] \sinh[\text{Integral}[d (x - \#1)] + \right. \\ & \quad \left. \sinh[c + d \#1] \sinh[\text{Integral}[d (x - \#1)] + d \cosh[c + d \#1] \cosh[\text{Integral}[d (x - \#1)] \#1 + \right. \\ & \quad \left. d \cosh[\text{Integral}[d (x - \#1)] \sinh[c + d \#1] \#1 + d \cosh[c + d \#1] \right. \\ & \quad \left. \sinh[\text{Integral}[d (x - \#1)] \#1 + d \sinh[c + d \#1] \sinh[\text{Integral}[d (x - \#1)] \#1] \&] \right) \right) \end{aligned}$$

Problem 103: Result is not expressed in closed-form.

$$\int \frac{x^2 \cosh[c + d x]}{(a + b x^3)^2} dx$$

Optimal (type 4, 373 leaves, 12 steps) :

$$\begin{aligned} & -\frac{\cosh[c + d x]}{3 b (a + b x^3)} + \frac{d \cosh[\text{Integral}[\frac{a^{1/3} d}{b^{1/3}} + d x] \sinh[c - \frac{a^{1/3} d}{b^{1/3}}]}{9 a^{2/3} b^{4/3}} - \\ & \frac{(-1)^{1/3} d \cosh[\text{Integral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x] \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}]}{9 a^{2/3} b^{4/3}} + \\ & \frac{(-1)^{2/3} d \cosh[\text{Integral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x] \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}]}{9 a^{2/3} b^{4/3}} + \\ & \frac{(-1)^{1/3} d \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \sinh[\text{Integral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{9 a^{2/3} b^{4/3}} + \\ & \frac{d \cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \sinh[\text{Integral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{9 a^{2/3} b^{4/3}} + \\ & \frac{(-1)^{2/3} d \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \sinh[\text{Integral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x]}{9 a^{2/3} b^{4/3}} \end{aligned}$$

Result (type 7, 203 leaves) :

$$\begin{aligned} & \frac{1}{18 b^2} \left(-\frac{6 b \cosh[c + d x]}{a + b x^3} - d \operatorname{RootSum}[a + b \#1^3 \&, \right. \\ & \quad \left. \frac{1}{\#1^2} (\cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] - \operatorname{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] - \right. \\ & \quad \left. \cosh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] + \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)]) \& \right) + \\ & d \operatorname{RootSum}[a + b \#1^3 \&, \frac{1}{\#1^2} (\cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] + \\ & \quad \left. \cosh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] \sinh[c + d \#1] + \cosh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] + \right. \\ & \quad \left. \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)]) \& \right) \end{aligned}$$

Problem 104: Result is not expressed in closed-form.

$$\int \frac{x \cosh[c + d x]}{(a + b x^3)^2} dx$$

Optimal (type 4, 695 leaves, 34 steps):

$$\begin{aligned} & \frac{\cosh[c + d x]}{3 a b x} - \frac{\cosh[c + d x]}{3 b x (a + b x^3)} - \frac{(-1)^{2/3} \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{9 a^{4/3} b^{2/3}} + \\ & \frac{(-1)^{1/3} \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x]}{9 a^{4/3} b^{2/3}} - \\ & \frac{\cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{9 a^{4/3} b^{2/3}} - \frac{d \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x] \sinh[c - \frac{a^{1/3} d}{b^{1/3}}]}{9 a b} - \\ & \frac{d \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x] \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}]}{9 a b} - \\ & \frac{d \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x] \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}]}{9 a b} + \\ & \frac{d \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{9 a b} + \\ & \frac{(-1)^{2/3} \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{9 a^{4/3} b^{2/3}} - \\ & \frac{d \cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{9 a b} - \frac{\sinh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{9 a^{4/3} b^{2/3}} - \\ & \frac{d \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x]}{9 a b} + \\ & \frac{(-1)^{1/3} \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x]}{9 a^{4/3} b^{2/3}} \end{aligned}$$

Result (type 7, 387 leaves) :

$$\frac{1}{18 a b (a + b x^3)} \left(6 b x^2 \cosh[c + d x] + (a + b x^3) \operatorname{RootSum}[a + b \#1^3 \&, \frac{1}{\#1} (\cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] - \cosh[\cosh[c + d \#1] \sinh[d (x - \#1)]] + \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] + d \cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] \#1 - d \operatorname{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] \#1 - d \cosh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] \#1 + d \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] \#1) \&] - (a + b x^3) \operatorname{RootSum}[a + b \#1^3 \&, \frac{1}{\#1} (-\cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] - \cosh[\cosh[c + d \#1] \sinh[d (x - \#1)]] - \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] + d \cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] \#1 + d \operatorname{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] \#1 + d \cosh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] \#1 + d \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] \#1) \&] \right)$$

Problem 105: Result is not expressed in closed-form.

$$\int \frac{\cosh[c + d x]}{(a + b x^3)^2} dx$$

Optimal (type 4, 739 leaves, 36 steps) :

$$\begin{aligned}
& \frac{\cosh[c + d x]}{3 a b x^2} - \frac{\cosh[c + d x]}{3 b x^2 (a + b x^3)} - \frac{2 (-1)^{1/3} \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{9 a^{5/3} b^{1/3}} + \\
& \frac{2 (-1)^{2/3} \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x]}{9 a^{5/3} b^{1/3}} + \\
& \frac{2 \cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{9 a^{5/3} b^{1/3}} + \frac{d \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x] \sinh[c - \frac{a^{1/3} d}{b^{1/3}}]}{9 a^{4/3} b^{2/3}} + \\
& \frac{(-1)^{2/3} d \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x] \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}]}{9 a^{4/3} b^{2/3}} - \\
& \frac{(-1)^{1/3} d \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x] \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}]}{9 a^{4/3} b^{2/3}} - \\
& \frac{(-1)^{2/3} d \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{9 a^{4/3} b^{2/3}} + \\
& \frac{2 (-1)^{1/3} \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{9 a^{5/3} b^{1/3}} + \\
& \frac{d \cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{9 a^{4/3} b^{2/3}} + \frac{2 \sinh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{9 a^{5/3} b^{1/3}} - \\
& \frac{(-1)^{1/3} d \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x]}{9 a^{4/3} b^{2/3}} + \\
& \frac{2 (-1)^{2/3} \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x]}{9 a^{5/3} b^{1/3}}
\end{aligned}$$

Result (type 7, 387 leaves):

$$\begin{aligned}
& \frac{1}{18 a b (a + b x^3)} \left(6 b x \cosh[c + d x] + \right. \\
& (a + b x^3) \operatorname{RootSum}[a + b \#1^3 \&, \frac{1}{\#1^2} \left(2 \cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] - \right. \\
& 2 \operatorname{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] - 2 \cosh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] + \\
& 2 \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] + d \cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] \\
& \#1 - d \operatorname{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] \#1 - d \cosh[c + d \#1] \\
& \operatorname{SinhIntegral}[d (x - \#1)] \#1 + d \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] \#1) \&] - \\
& (a + b x^3) \operatorname{RootSum}[a + b \#1^3 \&, \frac{1}{\#1^2} \left(-2 \cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] - \right. \\
& 2 \operatorname{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] - 2 \cosh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] - \\
& 2 \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] + d \cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] \\
& \#1 + d \operatorname{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] \#1 + d \cosh[c + d \#1] \\
& \operatorname{SinhIntegral}[d (x - \#1)] \#1 + d \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] \#1) \&] \left. \right)
\end{aligned}$$

Problem 106: Result more than twice size of optimal antiderivative.

$$\int \frac{\cosh[c + d x]}{x (a + b x^3)^2} dx$$

Optimal (type 4, 697 leaves, 41 steps):

$$\begin{aligned} & \frac{\cosh[c + d x]}{3 a b x^3} - \frac{\cosh[c + d x]}{3 b x^3 (a + b x^3)} + \frac{\cosh[c] \operatorname{CoshIntegral}[d x]}{a^2} - \\ & \frac{\cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{3 a^2} - \\ & \frac{\cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x]}{3 a^2} - \\ & \frac{\cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{3 a^2} - \frac{d \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x] \operatorname{Sinh}[c - \frac{a^{1/3} d}{b^{1/3}}]}{9 a^{5/3} b^{1/3}} + \\ & \frac{(-1)^{1/3} d \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x] \operatorname{Sinh}[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}]}{9 a^{5/3} b^{1/3}} - \\ & \frac{(-1)^{2/3} d \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x] \operatorname{Sinh}[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}]}{9 a^{5/3} b^{1/3}} + \\ & \frac{\operatorname{Sinh}[c] \operatorname{SinhIntegral}[d x]}{a^2} - \frac{(-1)^{1/3} d \operatorname{Cosh}[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{9 a^{5/3} b^{1/3}} + \\ & \frac{\operatorname{Sinh}[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{3 a^2} - \\ & \frac{d \operatorname{Cosh}[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{9 a^{5/3} b^{1/3}} - \frac{\operatorname{Sinh}[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{3 a^2} - \\ & \frac{(-1)^{2/3} d \operatorname{Cosh}[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x]}{9 a^{5/3} b^{1/3}} - \\ & \frac{\operatorname{Sinh}[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x]}{3 a^2} \end{aligned}$$

Result (type 4, 5530 leaves):

$$\begin{aligned} & \operatorname{Sinh}[c] \left(\frac{\operatorname{SinhIntegral}[d x]}{a^2} - \right. \\ & \left. \left((2 b^{1/3} - 3 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3}) \left(-\operatorname{CoshIntegral}[d \left(\frac{a^{1/3}}{b^{1/3}} + x \right)] \operatorname{Sinh}[\frac{a^{1/3} d}{b^{1/3}}] + \right. \right. \right. \\ & \left. \left. \left. \operatorname{Cosh}[\frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[d \left(\frac{a^{1/3}}{b^{1/3}} + x \right)] \right) \right) \Bigg/ \left((-1 + (-1)^{1/3}) (1 + (-1)^{1/3})^2 a^2 b^{1/3} \right) + \right. \end{aligned}$$

$$\begin{aligned}
& \left(\left(21 - 22 (-1)^{1/3} + 21 (-1)^{2/3} \right) b^{1/3} \left(-\frac{\text{Sinh}[dx]}{b^{1/3} \left(-(-1)^{1/3} a^{1/3} + b^{1/3} x \right)} + \frac{1}{b^{2/3}} \right. \right. \\
& \quad d \left(\text{Cosh}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \text{CoshIntegral}\left[-\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} + dx \right] - \right. \\
& \quad \left. \left. \text{Sinh}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \text{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx \right] \right) \right) / \\
& \left(3 \left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^{5/3} \right) + \left(\left(22 - 21 (-1)^{1/3} + 21 (-1)^{2/3} \right) b^{1/3} \right. \\
& \quad \left(-\frac{\text{Sinh}[dx]}{b^{1/3} \left(a^{1/3} + b^{1/3} x \right)} + \frac{1}{b^{2/3}} d \left(\text{Cosh}\left[\frac{a^{1/3} d}{b^{1/3}} \right] \text{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx \right] - \right. \right. \\
& \quad \left. \left. \text{Sinh}\left[\frac{a^{1/3} d}{b^{1/3}} \right] \text{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx \right] \right) \right) / \left(3 \left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^{5/3} \right) + \\
& \left(\left(22 b^{1/3} - 21 (-1)^{1/3} b^{1/3} + 21 (-1)^{2/3} b^{1/3} \right) \left(-\frac{\text{Sinh}[dx]}{b^{1/3} \left((-1)^{2/3} a^{1/3} + b^{1/3} x \right)} + \frac{1}{b^{2/3}} \right. \right. \\
& \quad d \left(\text{Cosh}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \text{CoshIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx \right] - \right. \\
& \quad \left. \left. \text{Sinh}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \text{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx \right] \right) \right) / \\
& \left(3 \left(1 + (-1)^{1/3} \right)^2 a^{5/3} \right) + \left(\text{i} \left(2 b^{1/3} - 3 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3} \right) \right. \\
& \quad \left(\text{CosIntegral}\left[-\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} + \text{i} dx \right] \text{Sin}\left[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} \right] - \right. \\
& \quad \left. \left. \text{Cos}\left[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} \right] \text{SinIntegral}\left[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} - \text{i} dx \right] \right) \right) / \\
& \left(\left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^2 b^{1/3} \right) + \left(\text{i} \left(3 b^{1/3} - 2 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3} \right) \right. \\
& \quad \left(\text{CosIntegral}\left[-\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} + \text{i} dx \right] \text{Sin}\left[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} \right] - \right. \\
& \quad \left. \left. \text{Cos}\left[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} \right] \text{SinIntegral}\left[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} - \text{i} dx \right] \right) \right) / \left(\left(1 + (-1)^{1/3} \right)^2 a^2 b^{1/3} \right) + \\
& \text{Cosh}[c] \left(\frac{\text{CoshIntegral}[dx]}{a^2} + \left(\left(22 - 21 (-1)^{1/3} + 21 (-1)^{2/3} \right) \right. \right. \\
& \quad \left. \left. - \frac{b^{1/3} \text{Cosh}[dx]}{a^{1/3} + b^{1/3} x} - d \text{CoshIntegral}\left[d \left(\frac{a^{1/3}}{b^{1/3}} + x \right) \right] \text{Sinh}\left[\frac{a^{1/3} d}{b^{1/3}} \right] + \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{\text{d} \operatorname{Cosh} \left[\frac{a^{1/3} d}{b^{1/3}} \right] \operatorname{SinhIntegral} \left[d \left(\frac{a^{1/3}}{b^{1/3}} + x \right) \right]}{b^{1/3}} \right) / \\
& \left(3 \left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^{5/3} b^{1/3} \right) + \left(\left(21 - 22 (-1)^{1/3} + 21 (-1)^{2/3} \right) b^{1/3} \right. \\
& \left(\frac{\operatorname{Cosh}[d x]}{b^{1/3} \left((-1)^{1/3} a^{1/3} - b^{1/3} x \right)} + \frac{1}{b^{2/3}} d \left(\operatorname{CoshIntegral} \left[d \left(-\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}} + x \right) \right] \operatorname{Sinh} \left[\right. \right. \right. \\
& \left. \left. \left. \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] - \operatorname{Cosh} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{SinhIntegral} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x \right] \right) \right) / \\
& \left(3 \left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^{5/3} \right) - \left(\left(2 b^{1/3} - 3 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3} \right) \right. \\
& \left(\operatorname{Cosh} \left[\frac{a^{1/3} d}{b^{1/3}} \right] \operatorname{CoshIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + d x \right] - \operatorname{Sinh} \left[\frac{a^{1/3} d}{b^{1/3}} \right] \operatorname{SinhIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + d x \right] \right) \right) / \\
& \left(\left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^2 b^{1/3} \right) + \\
& \left(\left(22 b^{1/3} - 21 (-1)^{1/3} b^{1/3} + 21 (-1)^{2/3} b^{1/3} \right) \left(-\frac{\operatorname{Cosh}[d x]}{b^{1/3} \left((-1)^{2/3} a^{1/3} + b^{1/3} x \right)} - \right. \right. \\
& \left. \left. \frac{1}{b^{2/3}} d \left(\operatorname{CoshIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right] \operatorname{Sinh} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] - \right. \right. \\
& \left. \left. \operatorname{Cosh} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{SinhIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right] \right) \right) \right) / \\
& \left(3 \left(1 + (-1)^{1/3} \right)^2 a^{5/3} \right) - \left(\left(2 b^{1/3} - 3 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3} \right) \right. \\
& \left(\operatorname{Cos} \left[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} \right] \operatorname{CosIntegral} \left[-\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} + i d x \right] + \right. \\
& \left. \left. \operatorname{Sin} \left[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral} \left[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} - i d x \right] \right) \right) / \\
& \left(\left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^2 b^{1/3} \right) - \left(\left(3 b^{1/3} - 2 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3} \right) \right. \\
& \left(\operatorname{Cos} \left[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} \right] \operatorname{CosIntegral} \left[-\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} + i d x \right] + \right. \\
& \left. \left. \operatorname{Sin} \left[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral} \left[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} - i d x \right] \right) \right) / \left(\left(1 + (-1)^{1/3} \right)^2 a^2 b^{1/3} \right) + \\
& \frac{1}{2} \left(-\operatorname{Cosh}[c] \left(\frac{\operatorname{SinhIntegral}[d x]}{a^2} - \left(\left(2 b^{1/3} - 3 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3} \right) \right. \right. \right. \\
& \left. \left. \left. - \operatorname{CoshIntegral} \left[d \left(\frac{a^{1/3}}{b^{1/3}} + x \right) \right] \operatorname{Sinh} \left[\frac{a^{1/3} d}{b^{1/3}} \right] + \right. \right. \right. \\
& \left. \left. \left. - \operatorname{Cosh} \left[d \left(\frac{a^{1/3}}{b^{1/3}} + x \right) \right] \operatorname{Sinh} \left[\frac{a^{1/3} d}{b^{1/3}} \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\cosh\left[\frac{a^{1/3} d}{b^{1/3}}\right] \sinh\text{Integral}\left[d\left(\frac{a^{1/3}}{b^{1/3}} + x\right)\right] \right) \Big/ \left(\left(-1 + (-1)^{1/3}\right) \left(1 + (-1)^{1/3}\right)^2 a^2 b^{1/3} \right) + \\
& \left(\left(21 - 22 (-1)^{1/3} + 21 (-1)^{2/3}\right) b^{1/3} \left(-\frac{\sinh[d x]}{b^{1/3} \left(-(-1)^{1/3} a^{1/3} + b^{1/3} x\right)} + \frac{1}{b^{2/3}} \right. \right. \\
& \left. \left. d \left(\cosh\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \cosh\text{Integral}\left[-\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} + d x\right] - \right. \right. \\
& \left. \left. \sinh\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \sinh\text{Integral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x\right] \right) \right) \Big/ \\
& \left(3 \left(-1 + (-1)^{1/3}\right) \left(1 + (-1)^{1/3}\right)^2 a^{5/3} \right) + \left(\left(22 - 21 (-1)^{1/3} + 21 (-1)^{2/3}\right) b^{1/3} \right. \\
& \left. \left(-\frac{\sinh[d x]}{b^{1/3} \left(a^{1/3} + b^{1/3} x\right)} + \frac{1}{b^{2/3}} d \left(\cosh\left[\frac{a^{1/3} d}{b^{1/3}}\right] \cosh\text{Integral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] - \sinh\left[\frac{a^{1/3} d}{b^{1/3}}\right] \right. \right. \\
& \left. \left. \sinh\text{Integral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x\right] \right) \right) \Big/ \left(3 \left(-1 + (-1)^{1/3}\right) \left(1 + (-1)^{1/3}\right)^2 a^{5/3} \right) + \\
& \left(\left(22 b^{1/3} - 21 (-1)^{1/3} b^{1/3} + 21 (-1)^{2/3} b^{1/3}\right) \left(-\frac{\sinh[d x]}{b^{1/3} \left((-1)^{2/3} a^{1/3} + b^{1/3} x\right)} + \frac{1}{b^{2/3}} \right. \right. \\
& \left. \left. d \left(\cosh\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \cosh\text{Integral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] - \sinh\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \right. \right. \\
& \left. \left. \sinh\text{Integral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x\right] \right) \right) \Big/ \left(3 \left(1 + (-1)^{1/3}\right)^2 a^{5/3} \right) + \\
& \left(\frac{1}{2} \left(2 b^{1/3} - 3 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3}\right) \left(\text{cosIntegral}\left[-\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} + \frac{1}{2} d x\right] \right. \right. \\
& \left. \left. \sin\left[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}}\right] - \cos\left[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}}\right] \text{sinIntegral}\left[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} - \frac{1}{2} d x\right] \right) \right) \Big/ \\
& \left(\left(-1 + (-1)^{1/3}\right) \left(1 + (-1)^{1/3}\right)^2 a^2 b^{1/3} \right) + \left(\frac{1}{2} \left(3 b^{1/3} - 2 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3}\right) \right. \\
& \left. \left(\text{cosIntegral}\left[-\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} + \frac{1}{2} d x\right] \sin\left[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}}\right] - \cos\left[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}}\right] \right. \right. \\
& \left. \left. \sinIntegral\left[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} - \frac{1}{2} d x\right] \right) \right) \Big/ \left(\left(1 + (-1)^{1/3}\right)^2 a^2 b^{1/3} \right) - \sinh[c] \\
& \left(\frac{\cosh\text{Integral}[d x]}{a^2} + \left(\left(22 - 21 (-1)^{1/3} + 21 (-1)^{2/3}\right) \left(-\frac{b^{1/3} \cosh[d x]}{a^{1/3} + b^{1/3} x} - d \cosh\text{Integral}\left[\right. \right. \right. \right. \\
& \left. \left. \left. \left. d \left(\frac{a^{1/3}}{b^{1/3}} + x\right) \right] \sinh\left[\frac{a^{1/3} d}{b^{1/3}}\right] + d \cosh\left[\frac{a^{1/3} d}{b^{1/3}}\right] \sinh\text{Integral}\left[d\left(\frac{a^{1/3}}{b^{1/3}} + x\right)\right] \right) \right) \Big/
\end{aligned}$$

$$\begin{aligned}
& \left(3 \left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^{5/3} b^{1/3} \right) + \left(\left(21 - 22 (-1)^{1/3} + 21 (-1)^{2/3} \right) b^{1/3} \right. \\
& \left. \left(\frac{\text{Cosh}[d x]}{b^{1/3} \left((-1)^{1/3} a^{1/3} - b^{1/3} x \right)} + \frac{1}{b^{2/3}} d \left(\text{CoshIntegral} \left[d \left(-\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}} + x \right) \right] \text{Sinh} \right. \right. \\
& \left. \left. \left(\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right) - \text{Cosh} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \text{SinhIntegral} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x \right] \right) \right) \right) / \\
& \left(3 \left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^{5/3} \right) - \left(\left(2 b^{1/3} - 3 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3} \right) \right. \\
& \left. \left(\text{Cosh} \left[\frac{a^{1/3} d}{b^{1/3}} \right] \text{CoshIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + d x \right] - \text{Sinh} \left[\frac{a^{1/3} d}{b^{1/3}} \right] \text{SinhIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + d x \right] \right) \right) / \\
& \left(\left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^2 b^{1/3} \right) + \left(\left(22 b^{1/3} - 21 (-1)^{1/3} b^{1/3} + 21 (-1)^{2/3} b^{1/3} \right) \right. \\
& \left. \left(-\frac{\text{Cosh}[d x]}{b^{1/3} \left((-1)^{2/3} a^{1/3} + b^{1/3} x \right)} - \frac{1}{b^{2/3}} d \left(\text{CoshIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right] \text{Sinh} \right. \right. \right. \\
& \left. \left. \left. \left(\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right) - \text{Cosh} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \text{SinhIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right] \right) \right) \right) / \\
& \left(3 \left(1 + (-1)^{1/3} \right)^2 a^{5/3} \right) - \left(\left(2 b^{1/3} - 3 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3} \right) \right. \\
& \left. \left(\text{Cos} \left[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} \right] \text{CosIntegral} \left[-\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} + \text{i} d x \right] + \right. \right. \\
& \left. \left. \text{Sin} \left[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} \right] \text{SinIntegral} \left[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} - \text{i} d x \right] \right) \right) / \\
& \left(\left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^2 b^{1/3} \right) - \left(\left(3 b^{1/3} - 2 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3} \right) \right. \\
& \left. \left(\text{Cos} \left[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} \right] \text{CosIntegral} \left[-\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} + \text{i} d x \right] + \text{Sin} \left[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} \right] \right. \right. \\
& \left. \left. \text{SinIntegral} \left[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} - \text{i} d x \right] \right) \right) / \left(\left(1 + (-1)^{1/3} \right)^2 a^2 b^{1/3} \right) + \\
& \frac{1}{2} \left(\text{Cosh}[c] \left(\frac{\text{SinhIntegral}[d x]}{a^2} - \left(\left(2 b^{1/3} - 3 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3} \right) \right. \right. \right. \\
& \left. \left. \left(-\text{CoshIntegral} \left[d \left(\frac{a^{1/3}}{b^{1/3}} + x \right) \right] \text{Sinh} \left[\frac{a^{1/3} d}{b^{1/3}} \right] + \right. \right. \right. \\
& \left. \left. \left. \text{Cosh} \left[\frac{a^{1/3} d}{b^{1/3}} \right] \text{SinhIntegral} \left[d \left(\frac{a^{1/3}}{b^{1/3}} + x \right) \right] \right) \right) / \left(\left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^2 b^{1/3} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(21 - 22 (-1)^{1/3} + 21 (-1)^{2/3} \right) b^{1/3} \left(- \frac{\operatorname{Sinh}[dx]}{b^{1/3} \left(-(-1)^{1/3} a^{1/3} + b^{1/3} x \right)} + \frac{1}{b^{2/3}} \right. \\
& d \left(\operatorname{Cosh} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{CoshIntegral} \left[- \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} + dx \right] - \right. \\
& \left. \left. \operatorname{Sinh} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{SinhIntegral} \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx \right] \right) \right) / \\
& \left(3 \left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^{5/3} \right) + \left(\left(22 - 21 (-1)^{1/3} + 21 (-1)^{2/3} \right) b^{1/3} \right. \\
& \left(- \frac{\operatorname{Sinh}[dx]}{b^{1/3} \left(a^{1/3} + b^{1/3} x \right)} + \frac{1}{b^{2/3}} d \left(\operatorname{Cosh} \left[\frac{a^{1/3} d}{b^{1/3}} \right] \operatorname{CoshIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + dx \right] - \operatorname{Sinh} \left[\frac{a^{1/3} d}{b^{1/3}} \right] \right. \right. \\
& \left. \left. \operatorname{SinhIntegral} \left[\frac{a^{1/3} d}{b^{1/3}} + dx \right] \right) \right) / \left(3 \left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^{5/3} \right) + \right. \\
& \left(\left(22 b^{1/3} - 21 (-1)^{1/3} b^{1/3} + 21 (-1)^{2/3} b^{1/3} \right) \left(- \frac{\operatorname{Sinh}[dx]}{b^{1/3} \left((-1)^{2/3} a^{1/3} + b^{1/3} x \right)} + \frac{1}{b^{2/3}} \right. \right. \\
& d \left(\operatorname{Cosh} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \operatorname{CoshIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx \right] - \operatorname{Sinh} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \right. \\
& \left. \left. \operatorname{SinhIntegral} \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx \right] \right) \right) / \left(3 \left(1 + (-1)^{1/3} \right)^2 a^{5/3} \right) + \right. \\
& \left. \left(\frac{1}{2} \left(2 b^{1/3} - 3 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3} \right) \left(\operatorname{CosIntegral} \left[- \frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} + \frac{1}{2} dx \right] \right. \right. \right. \\
& \left. \left. \left. \operatorname{Sin} \left[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} \right] - \operatorname{Cos} \left[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral} \left[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} - \frac{1}{2} dx \right] \right) \right) / \right. \\
& \left(\left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^2 b^{1/3} \right) + \left(\frac{1}{2} \left(3 b^{1/3} - 2 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3} \right) \right. \\
& \left(\operatorname{CosIntegral} \left[- \frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} + \frac{1}{2} dx \right] \operatorname{Sin} \left[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} \right] - \operatorname{Cos} \left[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} \right] \right. \\
& \left. \left. \operatorname{SinIntegral} \left[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} - \frac{1}{2} dx \right] \right) \right) / \left(\left(1 + (-1)^{1/3} \right)^2 a^2 b^{1/3} \right) + \operatorname{Sinh}[c] \\
& \left(\frac{\operatorname{CoshIntegral}[dx]}{a^2} + \left(\left(22 - 21 (-1)^{1/3} + 21 (-1)^{2/3} \right) \left(- \frac{b^{1/3} \operatorname{Cosh}[dx]}{a^{1/3} + b^{1/3} x} - d \operatorname{CoshIntegral} \right. \right. \right. \\
& d \left(\frac{a^{1/3}}{b^{1/3}} + x \right) \left. \operatorname{Sinh} \left[\frac{a^{1/3} d}{b^{1/3}} \right] + d \operatorname{Cosh} \left[\frac{a^{1/3} d}{b^{1/3}} \right] \operatorname{SinhIntegral} \left[d \left(\frac{a^{1/3}}{b^{1/3}} + x \right) \right] \right) \right) / \\
& \left(3 \left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^{5/3} b^{1/3} \right) + \left(\left(21 - 22 (-1)^{1/3} + 21 (-1)^{2/3} \right) b^{1/3} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{\cosh[d x]}{b^{1/3} \left((-1)^{1/3} a^{1/3} - b^{1/3} x \right)} + \frac{1}{b^{2/3}} d \left(\operatorname{CoshIntegral}\left[d \left(-\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}} + x \right) \right] \sinh \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] - \cosh \left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} \right] \sinh \operatorname{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x \right] \right) \right) / \\
& \left(3 \left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^{5/3} \right) - \left(2 b^{1/3} - 3 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3} \right) \\
& \left(\cosh \left[\frac{a^{1/3} d}{b^{1/3}} \right] \operatorname{CoshIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x \right] - \sinh \left[\frac{a^{1/3} d}{b^{1/3}} \right] \operatorname{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + d x \right] \right) / \\
& \left(\left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^2 b^{1/3} \right) + \left(22 b^{1/3} - 21 (-1)^{1/3} b^{1/3} + 21 (-1)^{2/3} b^{1/3} \right) \\
& \left(- \frac{\cosh[d x]}{b^{1/3} \left((-1)^{2/3} a^{1/3} + b^{1/3} x \right)} - \frac{1}{b^{2/3}} d \left(\operatorname{CoshIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right] \sinh \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] - \cosh \left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} \right] \sinh \operatorname{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x \right] \right) \right) / \\
& \left(3 \left(1 + (-1)^{1/3} \right)^2 a^{5/3} \right) - \left(2 b^{1/3} - 3 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3} \right) \\
& \left(\cos \left[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} \right] \operatorname{CosIntegral}\left[-\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} + i d x \right] + \sin \left[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} \right] \operatorname{SinIntegral}\left[\frac{(-1)^{1/6} a^{1/3} d}{b^{1/3}} - i d x \right] \right) / \\
& \left(\left(-1 + (-1)^{1/3} \right) \left(1 + (-1)^{1/3} \right)^2 a^2 b^{1/3} \right) - \left(3 b^{1/3} - 2 (-1)^{1/3} b^{1/3} + 3 (-1)^{2/3} b^{1/3} \right) \\
& \left(\cos \left[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} \right] \operatorname{CosIntegral}\left[-\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} + i d x \right] + \sin \left[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} \right] \right. \\
& \left. \sin \operatorname{SinIntegral}\left[\frac{(-1)^{5/6} a^{1/3} d}{b^{1/3}} - i d x \right] \right) / \left(\left(1 + (-1)^{1/3} \right)^2 a^2 b^{1/3} \right)
\end{aligned}$$

Problem 107: Result is not expressed in closed-form.

$$\int \frac{x^5 \cosh[c + d x]}{(a + b x^3)^3} dx$$

Optimal (type 4, 784 leaves, 36 steps):

$$\begin{aligned}
& -\frac{x^3 \cosh[c + d x]}{6 b (a + b x^3)^2} - \frac{\cosh[c + d x]}{6 b^2 (a + b x^3)} - \frac{(-1)^{2/3} d^2 \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{54 a^{1/3} b^{8/3}} + \\
& \frac{(-1)^{1/3} d^2 \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x]}{54 a^{1/3} b^{8/3}} - \\
& \frac{d^2 \cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{54 a^{1/3} b^{8/3}} + \frac{2 d \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x] \sinh[c - \frac{a^{1/3} d}{b^{1/3}}]}{27 a^{2/3} b^{7/3}} - \\
& \frac{2 (-1)^{1/3} d \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x] \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}]}{27 a^{2/3} b^{7/3}} + \\
& \frac{2 (-1)^{2/3} d \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x] \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}]}{27 a^{2/3} b^{7/3}} - \\
& \frac{d x \sinh[c + d x]}{18 b^2 (a + b x^3)} + \frac{2 (-1)^{1/3} d \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{27 a^{2/3} b^{7/3}} + \\
& \frac{(-1)^{2/3} d^2 \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{54 a^{1/3} b^{8/3}} + \\
& \frac{2 d \cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{27 a^{2/3} b^{7/3}} - \frac{d^2 \sinh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{54 a^{1/3} b^{8/3}} + \\
& \frac{2 (-1)^{2/3} d \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x]}{27 a^{2/3} b^{7/3}} + \\
& \frac{(-1)^{1/3} d^2 \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x]}{54 a^{1/3} b^{8/3}}
\end{aligned}$$

Result (type 7, 397 leaves):

$$\frac{1}{108 b^3} \left(d \operatorname{RootSum}[a + b \#1^3 \&, \right. \\ \left. \frac{1}{\#1^2} (-4 \cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] + 4 \operatorname{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] + \right. \\ \left. 4 \cosh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] - 4 \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] + \right. \\ \left. d \cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] \#1 - \right. \\ \left. d \operatorname{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] \#1 - d \cosh[c + d \#1] \right. \\ \left. \operatorname{SinhIntegral}[d (x - \#1)] \#1 + d \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] \#1) \&] + \right. \\ d \operatorname{RootSum}[a + b \#1^3 \&, \frac{1}{\#1^2} (4 \cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] + \right. \\ \left. 4 \operatorname{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] + 4 \cosh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] + \right. \\ \left. 4 \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] + d \cosh[c + d \#1] \operatorname{CoshIntegral}[d (x - \#1)] \right. \\ \left. \#1 + d \operatorname{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] \#1 + d \cosh[c + d \#1] \right. \\ \left. \operatorname{SinhIntegral}[d (x - \#1)] \#1 + d \sinh[c + d \#1] \operatorname{SinhIntegral}[d (x - \#1)] \#1) \&] - \right. \\ \left. 6 b (3 (a + 2 b x^3) \cosh[c + d x] + d x (a + b x^3) \sinh[c + d x]) \right) \\ \left. (a + b x^3)^2 \right)$$

Problem 108: Result is not expressed in closed-form.

$$\int \frac{x^4 \cosh[c + d x]}{(a + b x^3)^3} dx$$

Optimal (type 4, 1105 leaves, 47 steps):

$$\frac{\cosh[c + d x]}{9 a b^2 x} - \frac{x^2 \cosh[c + d x]}{6 b (a + b x^3)^2} - \frac{\cosh[c + d x]}{9 b^2 x (a + b x^3)} - \\ \frac{(-1)^{2/3} \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{27 a^{4/3} b^{5/3}} - \\ \frac{(-1)^{1/3} d^2 \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{54 a^{2/3} b^{7/3}} + \\ \frac{(-1)^{1/3} \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x]}{27 a^{4/3} b^{5/3}} + \\ \frac{(-1)^{2/3} d^2 \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x]}{54 a^{2/3} b^{7/3}} - \\ \frac{\cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{27 a^{4/3} b^{5/3}} + \frac{d^2 \cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{54 a^{2/3} b^{7/3}} - \\ \frac{d \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x] \sinh[c - \frac{a^{1/3} d}{b^{1/3}}]}{27 a b^2} - \\ \frac{d \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x] \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}]}{27 a b^2}$$

$$\begin{aligned}
& \frac{d \operatorname{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - dx\right] \operatorname{Sinh}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{27 a b^2} - \\
& \frac{d \operatorname{Sinh}\left[c + dx\right]}{18 b^2 (a + b x^3)} + \frac{d \operatorname{Cosh}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]}{27 a b^2} + \\
& \frac{(-1)^{2/3} \operatorname{Sinh}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]}{27 a^{4/3} b^{5/3}} + \\
& \frac{(-1)^{1/3} d^2 \operatorname{Sinh}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]}{54 a^{2/3} b^{7/3}} - \\
& \frac{d \operatorname{Cosh}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right]}{27 a b^2} - \frac{\operatorname{Sinh}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right]}{27 a^{4/3} b^{5/3}} + \\
& \frac{d^2 \operatorname{Sinh}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right]}{54 a^{2/3} b^{7/3}} - \\
& \frac{d \operatorname{Cosh}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right]}{27 a b^2} + \\
& \frac{(-1)^{1/3} \operatorname{Sinh}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right]}{27 a^{4/3} b^{5/3}} + \\
& \frac{(-1)^{2/3} d^2 \operatorname{Sinh}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right]}{54 a^{2/3} b^{7/3}}
\end{aligned}$$

Result (type 7, 675 leaves) :

$$\frac{1}{108 a b^3} \left(\text{RootSum}[a + b \#1^3 \&, \frac{1}{\#1^2} \left(a d^2 \cosh[c + d \#1] \text{CoshIntegral}[d (x - \#1)] - a d^2 \text{CoshIntegral}[d (x - \#1)] \right. \right.$$

$$\left. \left. d (x - \#1) \right] \sinh[c + d \#1] - a d^2 \cosh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] + a d^2 \sinh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] + 2 b \cosh[c + d \#1] \right. \\ \left. \text{CoshIntegral}[d (x - \#1)] \right) \#1 - 2 b \cosh[c + d \#1] \text{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] \#1 - 2 b \cosh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \#1 + 2 b \sinh[c + d \#1] \\ \text{SinhIntegral}[d (x - \#1)] \#1 + 2 b d \cosh[c + d \#1] \text{CoshIntegral}[d (x - \#1)] \#1^2 - 2 b d \cosh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \#1^2 + 2 b d \sinh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \#1^2) \& \right] - \\ \text{RootSum}[a + b \#1^3 \&, \frac{1}{\#1^2} \left(-a d^2 \cosh[c + d \#1] \text{CoshIntegral}[d (x - \#1)] - a d^2 \text{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] - a d^2 \cosh[c + d \#1] \right. \\ \left. \sinh[c + d \#1] - a d^2 \sinh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] - 2 b \cosh[c + d \#1] \text{CoshIntegral}[d (x - \#1)] \#1 - 2 b \cosh[c + d \#1] \right. \\ \left. \sinh[c + d \#1] \#1 - 2 b \cosh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \#1 - 2 b \sinh[c + d \#1] \right. \\ \left. \sinh[c + d \#1] \#1 + 2 b d \cosh[c + d \#1] \text{CoshIntegral}[d (x - \#1)] \#1^2 - 2 b d \cosh[c + d \#1] \right. \\ \left. \text{SinhIntegral}[d (x - \#1)] \#1^2 + 2 b d \sinh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \#1^2) \& \right] + \\ \frac{6 b \cosh[d x] (b x^2 (-a + 2 b x^3) \cosh[c] - a d (a + b x^3) \sinh[c])}{(a + b x^3)^2} + \\ \left. \frac{6 b (-a d (a + b x^3) \cosh[c] + b x^2 (-a + 2 b x^3) \sinh[c]) \sinh[d x]}{(a + b x^3)^2} \right)$$

Problem 109: Result is not expressed in closed-form.

$$\int \frac{x^3 \cosh[c + d x]}{(a + b x^3)^3} dx$$

Optimal (type 4, 776 leaves, 71 steps):

$$\begin{aligned}
& \frac{\cosh[c + d x]}{18 a b^2 x^2} - \frac{x \cosh[c + d x]}{6 b (a + b x^3)^2} - \frac{\cosh[c + d x]}{18 b^2 x^2 (a + b x^3)} - \\
& \frac{(-1)^{1/3} \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{27 a^{5/3} b^{4/3}} - \\
& \frac{d^2 \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{54 a b^2} + \\
& \frac{(-1)^{2/3} \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x]}{27 a^{5/3} b^{4/3}} - \\
& \frac{d^2 \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x]}{54 a b^2} + \\
& \frac{\cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{27 a^{5/3} b^{4/3}} - \frac{d^2 \cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{54 a b^2} + \\
& \frac{d \sinh[c + d x]}{18 a b^2 x} - \frac{d \sinh[c + d x]}{18 b^2 x (a + b x^3)} + \frac{(-1)^{1/3} \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{27 a^{5/3} b^{4/3}} + \\
& \frac{d^2 \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{54 a b^2} + \\
& \frac{\sinh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{27 a^{5/3} b^{4/3}} - \frac{d^2 \sinh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{54 a b^2} + \\
& \frac{(-1)^{2/3} \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x]}{27 a^{5/3} b^{4/3}} - \\
& \frac{d^2 \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x]}{54 a b^2}
\end{aligned}$$

Result (type 7, 429 leaves) :

$$\begin{aligned}
& -\frac{1}{108 a b^2} \left(\text{RootSum}[a + b \#1^3 \&, \right. \\
& \quad \frac{1}{\#1^2} (-2 \cosh[c + d \#1] \text{CoshIntegral}[d (x - \#1)] + 2 \text{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] + \\
& \quad 2 \cosh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] - 2 \sinh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] + \\
& \quad d^2 \cosh[c + d \#1] \text{CoshIntegral}[d (x - \#1)] \#1^2 - \\
& \quad d^2 \text{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] \#1^2 - d^2 \cosh[c + d \#1] \\
& \quad \text{SinhIntegral}[d (x - \#1)] \#1^2 + d^2 \sinh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \#1^2) \&] + \\
& \quad \text{RootSum}[a + b \#1^3 \&, \frac{1}{\#1^2} (-2 \cosh[c + d \#1] \text{CoshIntegral}[d (x - \#1)] - \\
& \quad 2 \text{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] - 2 \cosh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] - \\
& \quad 2 \sinh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] + d^2 \cosh[c + d \#1] \text{CoshIntegral}[d (x - \#1)] \\
& \quad \#1^2 + d^2 \text{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] \#1^2 + d^2 \cosh[c + d \#1] \\
& \quad \text{SinhIntegral}[d (x - \#1)] \#1^2 + d^2 \sinh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \#1^2) \&] - \\
& \quad \left. \frac{6 b x ((-2 a + b x^3) \cosh[c + d x] + d x (a + b x^3) \sinh[c + d x])}{(a + b x^3)^2} \right)
\end{aligned}$$

Problem 110: Result is not expressed in closed-form.

$$\int \frac{x^2 \cosh[c + d x]}{(a + b x^3)^3} dx$$

Optimal (type 4, 781 leaves, 37 steps):

$$\begin{aligned}
& -\frac{\cosh[c + d x]}{6 b (a + b x^3)^2} + \frac{(-1)^{2/3} d^2 \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{54 a^{4/3} b^{5/3}} - \\
& \frac{(-1)^{1/3} d^2 \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x]}{54 a^{4/3} b^{5/3}} + \\
& \frac{d^2 \cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{54 a^{4/3} b^{5/3}} + \frac{d \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x] \sinh[c - \frac{a^{1/3} d}{b^{1/3}}]}{27 a^{5/3} b^{4/3}} - \\
& \frac{(-1)^{1/3} d \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x] \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}]}{27 a^{5/3} b^{4/3}} + \\
& \frac{(-1)^{2/3} d \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x] \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}]}{27 a^{5/3} b^{4/3}} + \frac{d \sinh[c + d x]}{18 a b^2 x^2} - \\
& \frac{d \sinh[c + d x]}{18 b^2 x^2 (a + b x^3)} + \frac{(-1)^{1/3} d \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{27 a^{5/3} b^{4/3}} - \\
& \frac{(-1)^{2/3} d^2 \sinh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{54 a^{4/3} b^{5/3}} + \\
& \frac{d \cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{27 a^{5/3} b^{4/3}} + \frac{d^2 \sinh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{54 a^{4/3} b^{5/3}} + \\
& \frac{(-1)^{2/3} d \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x]}{27 a^{5/3} b^{4/3}} - \\
& \frac{(-1)^{1/3} d^2 \sinh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{SinhIntegral}[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + d x]}{54 a^{4/3} b^{5/3}}
\end{aligned}$$

Result (type 7, 423 leaves):

$$\begin{aligned}
& -\frac{1}{108 a b^2} \\
& \left(d \operatorname{RootSum}[a+b \#1^3 \&, \frac{1}{\#1^2} (2 \cosh[c+d \#1] \operatorname{CoshIntegral}[d (x-\#1)] - 2 \operatorname{CoshIntegral}[d (x-\#1)] \sinh[c+d \#1] - 2 \cosh[c+d \#1] \operatorname{SinhIntegral}[d (x-\#1)] + 2 \sinh[c+d \#1] \operatorname{SinhIntegral}[d (x-\#1)] + d \cosh[c+d \#1] \operatorname{CoshIntegral}[d (x-\#1)] \#1 - d \operatorname{CoshIntegral}[d (x-\#1)] \sinh[c+d \#1] \#1 - d \cosh[c+d \#1] \operatorname{SinhIntegral}[d (x-\#1)] \#1 + d \sinh[c+d \#1] \operatorname{SinhIntegral}[d (x-\#1)] \#1) \& + d \operatorname{RootSum}[a+b \#1^3 \&, \frac{1}{\#1^2} (-2 \cosh[c+d \#1] \operatorname{CoshIntegral}[d (x-\#1)] - 2 \cosh[c+d \#1] \operatorname{CoshIntegral}[d (x-\#1)] \sinh[c+d \#1] - 2 \cosh[c+d \#1] \operatorname{SinhIntegral}[d (x-\#1)] - 2 \sinh[c+d \#1] \operatorname{SinhIntegral}[d (x-\#1)] + d \cosh[c+d \#1] \operatorname{CoshIntegral}[d (x-\#1)] \#1 + d \operatorname{CoshIntegral}[d (x-\#1)] \sinh[c+d \#1] \#1 + d \cosh[c+d \#1] \operatorname{SinhIntegral}[d (x-\#1)] \#1 + d \sinh[c+d \#1] \operatorname{SinhIntegral}[d (x-\#1)] \#1) \& - 6 b \cosh[d x] (-3 a \cosh[c] + d x (a+b x^3) \sinh[c]) \over (a+b x^3)^2 - 6 b (d x (a+b x^3) \cosh[c] - 3 a \sinh[c]) \sinh[d x] \over (a+b x^3)^2 \right)
\end{aligned}$$

Problem 111: Result is not expressed in closed-form.

$$\int \frac{x \cosh[c + d x]}{(a + b x^3)^3} dx$$

Optimal (type 4, 1147 leaves, 89 steps):

$$\begin{aligned}
& -\frac{\cosh[c+d x]}{18 a^2 b^2 x^4} + \frac{2 \cosh[c+d x]}{9 a^2 b x} - \frac{\cosh[c+d x]}{6 b x (a+b x^3)^2} + \frac{\cosh[c+d x]}{18 b^2 x^4 (a+b x^3)} - \\
& \frac{2 (-1)^{2/3} \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{27 a^{7/3} b^{2/3}} + \\
& \frac{(-1)^{1/3} d^2 \cosh[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - d x]}{54 a^{5/3} b^{4/3}} + \\
& \frac{2 (-1)^{1/3} \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x]}{27 a^{7/3} b^{2/3}} - \frac{1}{54 a^{5/3} b^{4/3}} \\
& (-1)^{2/3} d^2 \cosh[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - d x] - \\
& \frac{2 \cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{27 a^{7/3} b^{2/3}} - \frac{d^2 \cosh[c - \frac{a^{1/3} d}{b^{1/3}}] \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x]}{54 a^{5/3} b^{4/3}} - \\
& \frac{2 d \operatorname{CoshIntegral}[\frac{a^{1/3} d}{b^{1/3}} + d x] \sinh[c - \frac{a^{1/3} d}{b^{1/3}}]}{27 a^2 b}
\end{aligned}$$

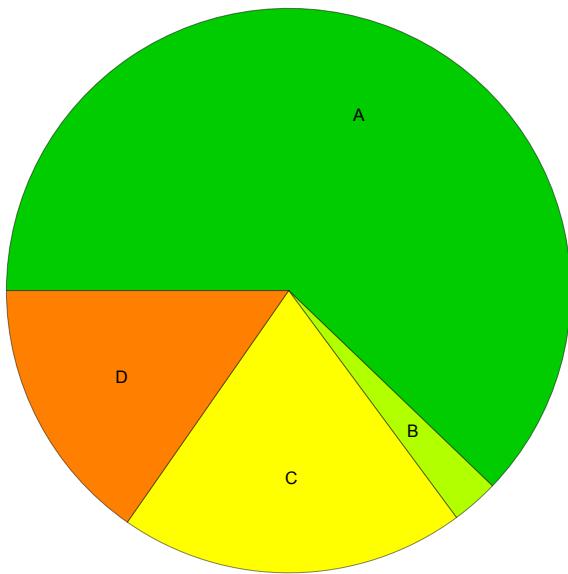
$$\begin{aligned}
& \frac{2 d \operatorname{CoshIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right] \operatorname{Sinh}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right]}{27 a^2 b} - \\
& \frac{2 d \operatorname{CoshIntegral}\left[-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - dx\right] \operatorname{Sinh}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right]}{27 a^2 b} + \frac{d \operatorname{Sinh}\left[c + dx\right]}{18 a b^2 x^3} - \\
& \frac{d \operatorname{Sinh}\left[c + dx\right]}{18 b^2 x^3 (a + b x^3)} + \frac{2 d \operatorname{Cosh}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]}{27 a^2 b} + \\
& \frac{2 (-1)^{2/3} \operatorname{Sinh}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]}{27 a^{7/3} b^{2/3}} - \\
& \frac{(-1)^{1/3} d^2 \operatorname{Sinh}\left[c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right]}{54 a^{5/3} b^{4/3}} - \\
& \frac{2 d \operatorname{Cosh}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right]}{27 a^2 b} - \\
& \frac{2 \operatorname{Sinh}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right]}{27 a^{7/3} b^{2/3}} - \frac{d^2 \operatorname{Sinh}\left[c - \frac{a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{a^{1/3} d}{b^{1/3}} + dx\right]}{54 a^{5/3} b^{4/3}} - \\
& \frac{2 d \operatorname{Cosh}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right]}{27 a^2 b} + \\
& \frac{2 (-1)^{1/3} \operatorname{Sinh}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right]}{27 a^{7/3} b^{2/3}} - \\
& \frac{(-1)^{2/3} d^2 \operatorname{Sinh}\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \operatorname{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right]}{54 a^{5/3} b^{4/3}}
\end{aligned}$$

Result (type 7, 669 leaves) :

$$\begin{aligned}
& \frac{1}{108 a^2 b^2} \\
& \left(\text{RootSum}[a + b \#1^3 \&, \frac{1}{\#1^2} (-a d^2 \cosh[c + d \#1] \text{CoshIntegral}[d (x - \#1)] + a d^2 \text{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] - a d^2 \sinh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] + 4 b \cosh[c + d \#1] \text{CoshIntegral}[d (x - \#1)] \#1 - 4 b \cosh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \#1 + 4 b \sinh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \#1 + 4 b d \cosh[c + d \#1] \text{CoshIntegral}[d (x - \#1)] \#1^2 - 4 b d \cosh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \#1^2 + 4 b d \sinh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \#1^2) \&] - \right. \\
& \text{RootSum}[a + b \#1^3 \&, \frac{1}{\#1^2} (a d^2 \cosh[c + d \#1] \text{CoshIntegral}[d (x - \#1)] + a d^2 \text{CoshIntegral}[d (x - \#1)] \sinh[c + d \#1] + \sinh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] + a d^2 \sinh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] - 4 b \cosh[c + d \#1] \text{CoshIntegral}[d (x - \#1)] \#1 - 4 b \cosh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \#1 - 4 b \sinh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \#1 + 4 b d \cosh[c + d \#1] \text{CoshIntegral}[d (x - \#1)] \#1^2 + 4 b d \cosh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \#1^2 + 4 b d \sinh[c + d \#1] \text{SinhIntegral}[d (x - \#1)] \#1^2) \&] + \\
& \frac{1}{(a + b x^3)^2} 6 b \cosh[d x] (b x^2 (7 a + 4 b x^3) \cosh[c] + a d (a + b x^3) \sinh[c]) + \\
& \frac{1}{(a + b x^3)^2} \\
& \left. b (a d (a + b x^3) \cosh[c] + b x^2 (7 a + 4 b x^3) \sinh[c]) \sinh[d x] \right)
\end{aligned}$$

Summary of Integration Test Results

111 integration problems



A - 69 optimal antiderivatives

B - 3 more than twice size of optimal antiderivatives

C - 22 unnecessarily complex antiderivatives

D - 17 unable to integrate problems

E - 0 integration timeouts